AGENDA

SENATE INTERIM COMMITTEE ON EDUCATION
and the
HOUSE INTERIM COMMITTEE ON EDUCATION

****

Thursday, January 14, 2010
1:30 P.M.
Room 151, State Capitol
Little Rock, Arkansas 72201

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Sen. Jimmy Jeffress, Chair
Sen. Steve Bryles, Vice Chair
Sen. Kim Hendren
Sen. Shane Broadway
Sen. Gene Jeffress
Sen. Mary Anne Salmon
Sen. Johnny Key
Sen. Joyce Elliott

Rep. Bill Abernathy, Chair
Rep. Nancy Duffy Blount, Vice Chair
Rep. Mark Martin
Rep. Rick Saunders
Rep. David Rainey
Rep. David Cook
Rep. Eddie Cheatham
Rep. Toni Bradford
Rep. Donna Hutchinson
Rep. Jerry Brown
Rep. Charolette Wagner
Rep. Steven Breedlove
Rep. Dan Greenberg
Rep. Tim Summers
Rep. Les "Skip" Carnine
Rep. Mark Perry

Rep. Linda Tyler
Rep. Robert Dale
Rep. Monty Betts
Rep. Jody Dickinson
Rep. Fred Allen, Non-Voting
Rep. Robert Moore, Non-Voting
Rep. Tommy Lee Baker, Non-Voting
Rep. Johnnie Roebuck, Non-Voting
Rep. Debra Hobbs, Non-Voting
Rep. Uvalde Lindsey, Non-Voting
Rep. Darrin Williams, Non-Voting
Rep. Karen Hopper, Non-Voting
Rep. Steve Cole, Non-Voting
Rep. Ann Clemmer, Non-Voting
Rep. Tiffany Rogers, Non-Voting
Rep. Mary Slinkard, Non-Voting

A. Call to Order – 1:30 P.M.

B. Approval of the Minutes of the December 17, 2009, Meeting **[Exhibit B]**

C. Discussion of the Proposed National Standards for K-12 Education and the Implications for Public Education in Arkansas **[Exhibits C-1 through C-4]**
Mr. Gene Wilhoit, Executive Director of the Council of Chief State School Officers and a former Director of the Arkansas Department of Education

D. Other Business

E. Adjournment

Notice: Silence your cell phones. Keep your personal conversations to a minimum. Observe restrictions designating areas as "*Members and Staff Only*"

EXHIBIT B

MINUTES

JOINT MEETING
OF THE
HOUSE AND SENATE INTERIM COMMITTEES ON EDUCATION

Thursday, December 17, 2009
10:00 A.M.
Room 171, State Capitol
Little Rock, Arkansas

Senator Jimmy Jeffress, the Chair of the Senate Interim Committee on Education, called the meeting to order at 10:00 a.m.

MEMBERS OF THE SENATE INTERIM COMMITTEE ON EDUCATION IN ATTENDANCE: Senator Jimmy Jeffress, Chair; Senator Steve Bryles, Vice-Chair; Senator Shane Broadway; Senator Joyce Elliott; Senator Kim Hendren; Senator Gene Jeffress; and Senator Mary Anne Salmon.

MEMBERS OF THE HOUSE INTERIM COMMITTEE ON EDUCATION IN ATTENDANCE: Representative Bill Abernathy, Chair; Representative Nancy Blount, Vice-Chair; Representative Monty Betts; Representative Toni Bradford; Representative Steve Breedlove; Representative Jerry Brown; Representative Les Carnine; Representative Eddie Cheatham; Representative Robert Dale; Representative Jody Dickinson; Representative Donna Hutchinson; Representative Mark Martin; Representative David Rainey; Representative Tim Summers; and Representative Charolette Wagner.

NON-VOTING MEMBERS OF THE HOUSE INTERIM COMMITTEE ON EDUCATION IN ATTENDANCE: Representative Fred Allen; Representative Tommy Lee Baker; Representative Debra Hobbs; Representative Karen Hopper; Representative Uvalde Lindsey; Representative Johnnie Roebuck; and Representative Tiffany Rogers.

OTHER MEMBERS OF THE GENERAL ASSEMBLY IN ATTENDANCE: Senator Steve Faris; Senator Randy Laverty; Senator Sue Madison; Senator David Wyatt; Representative Duncan Baird; Representative John Burris; Representative Richard Carroll; Representative Eddie Cooper; Representative Monty Davenport; Representative Otis Davis; Representative David Dunn; Representative Curren Everett; Representative Nathan George; Representative Buddy Lovell; Representative Barbara Nix; Representative George Overbey; and Representative Michael Patterson.

Without objection, the minutes of October 28, 2009, were approved as written.

Senator Jeffress announced that the Committees would deviate from the published agenda and discuss the interim study proposal (ISP) that comprised Agenda Item F.

Senator Hendren was recognized to discuss the ISP. He requested permission to first introduce a special guest, former Oklahoma State Representative Lance Cargill, who had served as the Speaker of the Oklahoma House of Representatives. Senator Jeffress welcomed former Representative Cargill and told him the Committees were honored to have him at the meeting.

Senator Hendren went on to discuss the rationale behind Interim Study Proposal 2009-203, REQUESTING THE SENATE COMMITTEE ON EDUCATION AND THE HOUSE COMMITTEE ON EDUCATION CONDUCT A STUDY OF THE PUBLIC MEETING FACILITIES PROVIDED FOR THE STATE BOARD OF EDUCATION MEETINGS.

Without objection, Interim Study Proposal 2009-203 by Senator Hendren, was adopted as written.

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Senator Jeffress announced that a new item would be added to the published agenda. A three-page memorial resolution, A MEMORIAL RESOLUTION HONORING JOSEPH KIRBY MAHONY, II, (1939-2009) FOR HIS EXTRAORDINARY SERVICE TO THE PEOPLE OF ARKANSAS, was distributed to the Committees. Senator Jeffress requested that the Committees adopt the resolution, which would be presented later in the meeting to Mr. Michael Mahony of New York, the son of the late Representative Mahony.

Without objection, the motion to adopt the Memorial Resolution was carried.

Senator Jeffress announced that the Committees would now take up Agenda Item C, Update on Implementation of the Education Plan of the Division of Youth Services, Arkansas Department of Human Services.

Senator Jeffress advised the Committees that this item was a follow-up on Representative Roebuck's legislation, Act 972 of 2009, An Act to Establish a System of Education Within the Division of Youth Services. He said that questions regarding funding remained unresolved.

Representative Roebuck was recognized. She noted the Arkansas Department of Education (ADE) and the Division of Youth Services (DYS) have had a successful collaboration. She thanked the Chair for this opportunity and looked forward to hearing the presentation.

Mr. Ron Angel, Director, Division of Youth Services, Arkansas Department of Human Services, was recognized. After introducing the DYS education staff; he discussed the establishment of an educational system within the DYS, utilizing a PowerPoint presentation. Mr. Angel discussed how implementation of the system can deliver opportunities such as:

- facilitating stakeholder ownership,
- providing a stronger system of education,
- improving teacher recruitment and retention, and,
- bridging the education gap for families in need.

Mr. Angel said that improved communications between the ADE and the DYS information technology systems would greatly improve the DYS ability:

- to transfer required reports,
- to build a better internal reporting system, and
- to enhance student records.

Mr. Brett Smith, Director of Education, Division of Youth Services, Arkansas Department of Human Services, was recognized. Mr. Smith gave a briefing on general education opportunities in the centers, outlining the school calendar which adheres to state guidelines. He also reported on the access students have to communication with families, and to vocational training.

Ms. Cindy Connell, Director of Curriculum Supervision and Instruction, Division of Youth Services, Arkansas Department of Human Services, was recognized. Ms. Connell talked about using the ADE-approved curriculum and textbooks in the education centers, and teaching the four core subjects, English, math, science and history, as well as other subjects. She spoke of utilizing summer programs for enrichment, credit recovery, and remediation, and talked about licensing of teachers in the system.

Ms. Mary Steel, Special Education Director, Division of Youth Services, Arkansas Department of Human Services, was recognized. Ms. Steel discussed:

- the continuum of services for students who receive special education services within the system.

EXHIBIT B

- using ARRA (American Recovery and Reinvestment Act) Plan funds to furnish special education classrooms under construction with technology equipment.
- developing due process training for special education teachers as it relates to state and federal rules and regulations.
- providing more planning time for teachers.

An extensive discussion of the various issues associated with the DYS system of education ensued, with the representatives of the DYS in attendance responding to various questions on:

- Availability of training programs for girls, creative programs, and college preparatory programs
- Opportunities for special education
- Adequacy and relevancy of textbooks
- Funding for improvements, library books and facilities, and updated technology
- Issuance and recognition of education center diplomas
- Teacher pay, insurance, and retirement benefits
- Transfer of responsibility for education center teachers to the state
- Cooperation of local school districts
- Counting an education center in the associated school district's funding report
- Tracking the effectiveness of the program
- Findings in the ADE study of the different centers in the system
- Improvement in classroom disruptions by students with behavioral problems
- Interface with alternative learning centers

Mr. Jerry Walsh, Executive Director, South Arkansas Youth Services, Inc. was recognized. Using a handout, Mr. Walsh summarized the number of students in the five facilities who are preparing to take the GED or preparing for college or for the military. Mr. Walsh was proud of one female juvenile from the Mansfield Center, now in college in Little Rock, who was awarded the Arkansas Coalition for Juvenile Justice Spirit Award, and was also awarded several small scholarships. He also spoke highly of the residential construction class at the Lewisville Center that worked with licensed contractors to help build a vocational classroom, and also erected a greenhouse on-site this past summer and fall.

Discussions followed on questions raised by the Committees about:

- Special support or scholarships for students because they have gone through a center
- Federal Pell Grant Program
- State of Arkansas support
- Identification of higher education colleges that have accepted students from the centers
- Student selection of a college based on geographical area
- Feedback from the colleges on student preparedness

Senator Jeffress thanked Mr. Angel and Mr. Walsh for their presentation.

Senator Jeffress announced that the Committees would return to the memorial resolution for Jodie Mahony and introduced **Mr. Michael Mahony**. At the request of Senator Jeffress, staff read the memorial resolution into the record. The resolution was presented to Mr. Mahony, who expressed his thanks and that of his family. Staff also presented Mr. Mahony the blue blazer always worn by his father. Senator Broadway was recognized and summed up that there will never be another Jodie Mahony, and that he meant a lot to all of the legislators and staff.

Update on Expenditure of Federal Funds for Education Purposes under the American Recovery and Reinvestment Act (ARRA) and other Federal Programs

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Mr. Chris Masingill, Arkansas Recovery Implementation Officer, Office of the Governor, was recognized. Mr. Masingill, using a PowerPoint presentation, provided the Committees with an overview of ARRA funding and discussed the importance of these funds in mitigating the impact of the current economic downturn and investing in the future economic development of the state.

Ms. Heather Gage, Special Advisor to the Commissioner, Arkansas Department of Education, was recognized. Ms. Gage, using a PowerPoint presentation, discussed the ARRA as it relates to K-12 education in Arkansas and spoke about three different areas:

- ARRA Formula Funds,
- ARRA Grant Opportunities, and
- Race to the Top.

Ms. Gage said that the United States Department of Education (USDOE) is focused on state success factors and wants to make sure that funding is being given to states that have the capacity to implement programs that enhance the effectiveness of their public education system. The USDOE wants to know that foundational education reforms have been enacted.

Senator Jeffress thanked Mr. Masingill and Ms. Gage for their presentations.

Update on the Arkansas Partnership for Teacher Quality (Ar-PTQ) Research Project

Mr. Richard Hutchinson, Government Relations/Instructional Issues Director, Arkansas Education Association (AEA), was recognized. Mr. Hutchinson presented an update on the collaborative partnership between the AEA, the National Education Association, and the American Association of Colleges for Teacher Education, which is looking at ways to strengthen teacher preparation in the state.

In order to give Committee members an idea of the breadth and depth of participation in the project, Mr. Hutchinson invited the national partners to introduce themselves:

Dr. Adriane Dorrington, National Education Association, Teacher Quality
Ms. Richelle Patterson, National Education Association, Teacher Quality
Kimberly T. Riley, Manager for State Programs and Policies, American Association of Colleges for Teacher Education
Carlene Anderson, Administrator, American Association of Colleges for Teacher Education

Then, he asked the Arkansas partners from teacher preparation institutions to introduce themselves:

Dr. Calvin Johnson, Dean, School of Education, University of Arkansas at Pine Bluff, and former Chair of the House Education Committee
Dr. Judy Harrison, Dean of Education, Teachers College, Henderson State University
Ms. Merribeth D. Bruning, Dean, Michael D. Huckabee School of Education and Director of Teacher Education, Ouachita Baptist University
Dr. Amanda Nolen, Assistant Professor, Department of Teacher Education, Teacher Education/Educational Foundations, University of Arkansas at Little Rock

Mr. Hutchinson concluded by saying that, following the initial study of identifying the data available in Arkansas, the next phase of the project is data review.

Dr. Angela Sewall, Dean, College of Education, University of Arkansas at Little Rock, was recognized. Dr. Sewall discussed the civic and economic impact of good teachers and good teacher preparation programs. She spoke of Arkansas's commitment to preparing high quality, fully-prepared teachers who can increase student achievement and school performance. Dr. Sewall said that Arkansas teacher preparation programs have to meet national standards; and increasing the number of college graduates is absolutely essential in this state. Dr. Sewall went on to describe the research process associated with this collaborative effort. She noted this research is indicative of the willingness of the state's teacher preparation programs to engage in meaningful self-examination, commitment to make bold improvements, and dedication to work with their K-12 partners to enhance Arkansas's educational and economic well-being.

Senator Jeffress thanked Mr. Hutchinson and Dr. Sewell for the update.

Senator Jeffress announced that the next meeting will be for the Joint Adequacy Evaluation Oversight Subcommittee on Tuesday, January 5, 2010, in Room 171 of the State Capitol.

There being no further business, the meeting adjourned at 12:21 p.m.

The Common Core State Standards Initiative is a significant and historic opportunity for states to collectively develop and adopt a core set of academic standards in mathematics and English language arts. Forty-eight states and three territories have joined the Common Core State Standards Initiative. The initiative is being jointly led by the NGA Center for Best Practices and the Council of Chief State School Officers in partnership with Achieve, ACT, and the College Board. It builds directly on recent efforts of leading organizations and states that have focused on developing college- and career-ready standards and ensures these standards are evidence- and research-based and internationally benchmarked to top-performing countries.

Why is this initiative important?

Currently, every state has its own set of academic standards, meaning public education students in each state are learning to different levels. All students must be prepared to compete with not only their American peers in the next state, but with students from around the world. If all 51 states and territories adopt the common core state standards, this initiative will affect 45.1 million students which is about 91 percent of the student population (Source: SchoolDataDirect.org; 2007).

Why is a common core of state standards good for students?

These standards will help prepare students with the knowledge and skills they need to succeed in college and careers and to be prepared to compete globally. Additionally, expectations for students will be consistent across all states and territories; this consistency will support students transitioning between states. Also, clearer standards will help students better understand what is expected of them and allow for more self-directed learning.

Why is a common core of state standards good for parents?

A common core of state standards will help parents understand what is expected of students and for college and work success. This understanding of what is expected of students will provide parents the opportunities to meaningfully engage in their children's education.

Why is a common core of state standards good for educators?

A common core of state standards will allow for more focused pre-service and professional development. Additionally, a common core will help assure that what is taught is aligned with assessments including formative, summative, and benchmarking. Also, educators will have the opportunity to tailor curriculum and teaching methods and promote the sharing of best practices.

Why is a common core of state standards good for states?

A common core of state standards will clearly articulate to parents, teachers, and the general public expectations for students. Shared standards will also help states better evaluate policy changes and identify best practices and needs for students and educators.

What is being produced and when?

A draft of the common core of state standards in mathematics and English language arts is available for public comment on www.corestandards.org. They are expected to be validated in November 2009. Additionally, in the winter of 2009/2010, the draft standards for grades K-12 will be released.

Common Core State Standards Initiative
Standards-Setting Criteria

The following criteria guided the standards development workgroups in setting the draft college and career readiness standards.

Preamble: The Common Core State Standards define the rigorous skills and knowledge in English Language Arts and Mathematics that need to be effectively taught and learned for students to be ready to succeed academically in credit-bearing, college-entry courses and in workforce training programs. These standards have been developed to be:

- Fewer, clearer, and higher, to best drive effective policy and practice;
- Aligned with college and work expectations, so that all students are prepared for success upon graduating from high school;
- Inclusive of rigorous content and applications of knowledge through higher-order skills, so that all students are prepared for the 21st century;
- Internationally benchmarked, so that all students are prepared for succeeding in our global economy and society; and
- Research and evidence-based.

The standards intend to set forward thinking goals for student performance based in evidence about what is required for success. The standards developed will set the stage for US education not just beyond next year, but for the next decade, and they must ensure ***all*** American students are prepared for the global economic workplace. Furthermore, the standards created will not lower the bar but raise it for all students; as such, we cannot narrow the college-ready focus of the standards to just preparation of students for college algebra and English composition and therefore will seek to ensure all students are prepared for all entry-level, credit-bearing, academic college courses in English, mathematics, the sciences, the social sciences, and the humanities. The objective is for all students to enter these classes ready for success (defined for these purposes as a C or better).

Goal: The standards as a whole must be essential, rigorous, clear and specific, coherent, and internationally benchmarked.

Essential: The standards must be reasonable in scope in defining the knowledge and skills students should have to be ready to succeed in entry-level, credit-bearing, academic college courses and in workforce training programs.

Workforce training programs pertain to careers that:
1) Offer competitive, livable salaries above the poverty line
2) Offer opportunities for career advancement
3) Are in a growing or sustainable industry

Common Core State Standards Initiative
Standards-Setting Considerations

The following considerations guided the standards development workgroups in setting the draft college and career readiness standards.

Fewer, clearer, higher: One of the goals of this process was to produce a set of fewer, clearer and higher standards. It is critical that any standards document be translatable to and teachable in the classroom. As such, the standards must cover only those areas that are critical for student success. This meant making tough decisions about what to include in the standards; however, these choices were important to ensure the standards are useable by teachers.

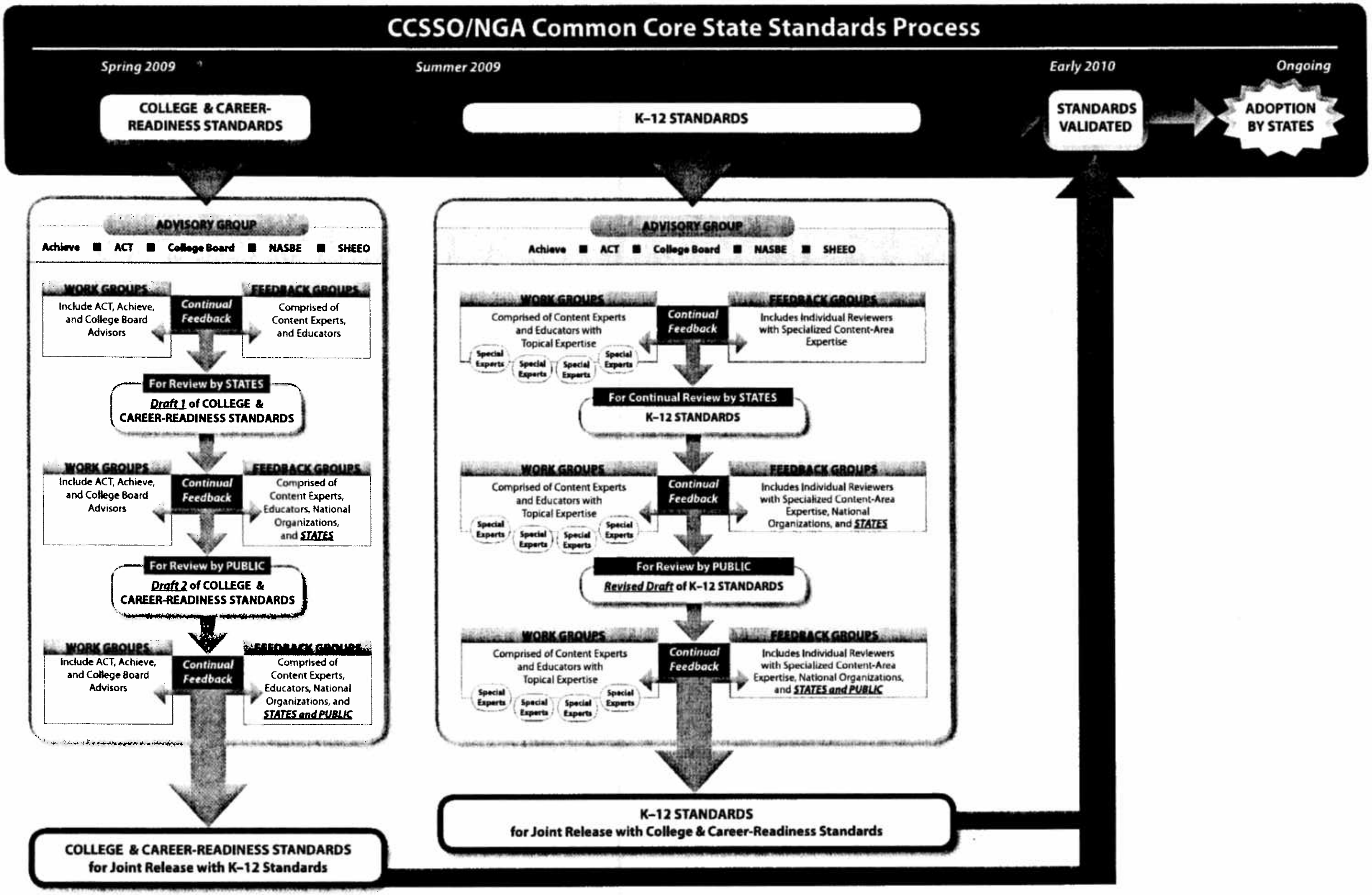
Evidence: This work has made unprecedented use of evidence in deciding what to include – or not include – in the standards. Each document includes a brief narrative on the choices that were made based on evidence. Rather than focusing on the *opinions* of experts exclusively, evidence to guide the decisions about what to include in the standards was used. This is a key difference between this process and the processes that have come before.

Internationally benchmarked: These standards are informed by the content, rigor and organization of standards of high-performing countries and states so that all students are prepared to succeed in a global economy and society.

Special populations: In the development of these standards, the inclusion of all types of learners was a priority. Writers selected language intended to make the standards documents accessible to different learners.

Assessment: While an assessment of the common core state standards in not currently being developed, these standards will ultimately be the basis for an assessment system that would include multiple measures of student performance. Once states agree on the final standards, attention will be turned to creating a high quality system of measurement that would include proper incentives for teachers to teach these standards and a variety of assessments that will reinforce teaching and learning tied to the agreed upon expectations.

Standards and curriculum: Standards are not curriculum. This initiative is about developing a set of standards that are common across states. The curriculum that follows will continue to be a local responsibility (or state-led, where appropriate). The curriculum could become more consistent from state to state based on the commonality of the standards; however, there are multiple ways to teach these standards, and therefore, there will be multiple approaches that could help students accomplish the goals set out in the standards.


CCSSO/NGA Common Core State Standards Process
Spring 2009
Summer 2009
Early 2010
Ongoing
COLLEGE & CAREER-READINESS STANDARDS
K-12 STANDARDS
STANDARDS VALIDATED
ADOPTION BY STATES
ADVISORY GROUP
Achieve ■ ACT ■ College Board ■ NASBE ■ SHEEO
WORK GROUPS
Include ACT, Achieve, and College Board Advisors
Continual Feedback
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Revised Draft of K-12 STANDARDS
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K-12 STANDARDS for Joint Release with College & Career-Readiness Standards

Core Standards for Reading, Writing, and Speaking and Listening

The Core Standards identify essential college- and career-ready skills and knowledge in reading, writing, and speaking and listening across the disciplines. While the English language arts classroom has often been seen as the proper site for literacy instruction, this document acknowledges that the responsibility for teaching such skills must also extend to the other content areas. Teachers in the social and natural sciences, the humanities, and mathematics need to use their content-area expertise to help students acquire the discipline-specific skills necessary to comprehend challenging texts and develop deep knowledge in those fields. At the same time, English language arts teachers not only must engage their students in a rich array of literature but also must help develop their students' ability to read complex works of nonfiction independently.

What is taught is just as important as how it is taught; the Core Standards should be accompanied by a comprehensive, content-rich curriculum. While this document defines the outcomes all students need to reach to be college and career ready, many important decisions about curriculum will necessarily be left to states, districts, schools, teachers, professional organizations, and parents. For example, while the standards require that students read texts of sufficient complexity, quality, and range, this document does not contain a required reading list. If states and districts choose to develop one, they should look at the Reading Exemplars provided here to get a sense of the level of complexity students must be able to handle independently when they read. Educators can also model their efforts on reading lists from around the nation and the world as long as the texts ultimately included meet the range and content standards in this document.

Standards today must ready students for competition and collaboration in a global, media-saturated environment. Colleges and universities have become international meetinghouses where people from across the globe learn with and from one another. At the same time, business today is truly a worldwide enterprise. Media-related technology helps shape what goes on in both college and the workplace; indeed, it has in some important ways reshaped the very nature of communication. Students who meet the Core Standards will have the reading, writing, speaking, and listening skills to flourish in the diverse, rapidly changing environments of college and careers.

Although reading, writing, and speaking and listening are articulated separately in the standards that follow, these divisions are made for the sake of clarity and manageability. In reality, the processes of communication are tightly interrelated and often reciprocal. The act of reading can no more be separated from the written word than the act of listening can be from the spoken word. When reading, students demonstrate their comprehension most commonly through a spoken or written interpretation of the text. As students solve problems, share insights, and build the knowledge they need for college and career success, they draw simultaneously on their capacities to read, write, speak, and listen.

Student Practices in Reading, Writing, and Speaking and Listening

The following practices in reading, writing, and speaking and listening undergird and help unify the rest of the standards document. They are the "premises"—broad statements about the nature of college and career readiness in reading, writing, and speaking and listening—that underlie the individual standards statements and cut across the various sections of the document. Every idea introduced here is subsequently represented in one or more places within the larger document.

* * *

Students who are college and career ready exhibit the following capacities in their reading, writing, and speaking and listening:

1. *They demonstrate independence as readers, writers, speakers, and listeners.*

   Students can, without significant scaffolding or support, comprehend and evaluate complex text across a range of types and disciplines, and they can construct effective arguments and clearly convey intricate or multifaceted information. Likewise, students are independently able to discern a speaker's key points as well as ask questions and articulate their own ideas.

2. *They build strong content knowledge.*

   Students build a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They demonstrate their ability to become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and the specific in-depth expertise needed to comprehend subject matter and solve problems in different fields. They refine their knowledge and share it through substantive writing and speaking.

3. *They respond to the varying demands of audience, task, purpose, and discipline.*

   Students consider their reading, writing, and speaking and listening in relation to the contextual factors of audience, task, purpose, and discipline. They appreciate nuances, such as how the composition and familiarity of the audience should affect tone. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in the natural sciences).

4. *They comprehend as well as critique.*

   Students are engaged and open-minded—but skeptical—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and assess the veracity of claims.

5. *They privilege evidence.*

   Students cite specific textual evidence when offering an oral or written interpretation of a piece of writing. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

6. *They care about precision.*

   Students are mindful of the impact of specific words and details, and they consider what would be achieved by different choices. Students pay especially close attention when precision matters most, such as in the case of reviewing significant data, making important distinctions, or analyzing a key moment in the action of a play or novel.

7. *They craft and look for structure.*

   Students attend to structure when organizing their own writing and speaking as well as when seeking to understand the work of others. They understand and make use of the ways of presenting information typical of different disciplines. They observe, for example, how authors of literary works craft the structure to unfold events and depict the setting.

8. *They use technology strategically and capably.*

   Students employ technology thoughtfully to enhance their reading, writing, speaking, and listening. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

Introductory Evidence Statement for Reading, Writing, Speaking and Listening Standards

To develop college- and career-ready standards for Reading, Writing, and Speaking and Listening that are rigorous, relevant, and internationally benchmarked, the work group consulted evidence from a wide array of sources. These included standards documents from high-performing states and nations; student performance data (including assessment scores and college grades); academic research; frameworks for assessments, such as NAEP; and results of surveys of postsecondary instructors and employers regarding what is most important for college and career readiness.

The evidence strongly suggests that similar reading, writing, speaking, and listening skills are necessary for success in both college and the workplace. A review of the standards of high-performing nations also suggests that many of these skills are already required in secondary schools internationally. The work group has endeavored to articulate these skills in the Core Standards, focusing educators, students, parents, and resources on what matters most.

Given that a set of standards cannot be simplistically "derived" from any body of evidence, the work group sometimes relied on reasoned judgment to interpret where the evidence was most compelling. For example, there is not a consensus among college faculty about the need for incoming students to be able to comprehend graphs, charts, and tables and to integrate information in these data displays with the information in the accompanying text. Although some evidence suggests that this skill is critical in the workplace and in some entry-level courses, college faculties from the various disciplines disagree on its value (with science and economics faculty rating it more highly than English and humanities professors do). The work group ultimately included a standard on the integration of text and data because the preponderance of the evidence suggests the skill's importance in meeting the demands of the twenty-first-century workplace and some college classrooms.

In most cases, the evidence is clearer. In writing, for example, there is unequivocal value placed on the logical progression of ideas. The expectation that high school graduates will be able to produce writing that is logical and coherent is found throughout the standards of top-performing countries and states. This ability is also valued highly by college faculty and employers. In response to such clear evidence, the work group included Writing student performance standard #5: "Create a logical progression of ideas or events and convey the relationships among them."

A bibliography of some of the sources we drew upon most is included at the end of this document. We also refer the reader to the Core Standards Web site (http://www.corestandards.org), which contains a similar bibliography (with links to full documents where available) as well as lists of standards linked to relevant sources of evidence.

Finally, while the standards reflect the best evidence available to date, the decisions the work group made are necessarily provisional. The core should be reexamined periodically as additional research on college and career readiness emerges. Indeed, this document may serve as an agenda for such research.

How to Read the Document

This document is divided into three main sections: strands, applications, and supporting materials.

Strands

There are three *strands*: Reading, Writing, and Speaking and Listening. Although each strand is presented discretely for ease of understanding, the document should be considered a coherent whole.

The three strands are each in turn divided into two sections: *Standards for Range and Content* and *Standards for Student Performance.*

Standards for Range and Content

The Standards for Range and Content in each strand describe the contexts in which college- and career-ready students must be able to read, write, speak, and listen. Rather than merely supplement or illustrate the numbered list of Standards for Student Performance, the Standards for Range and Content are themselves required and carry equal force.

Standards for Student Performance

The Standards for Student Performance in each strand enumerate the essential skills and understandings that students who are college and career ready in reading, writing, and speaking and listening must have no later than the end of high school.

Applications

The clearest examples of the integrated nature of communication are the *Applications of the Core* for Research and Media. The Core Standards for Reading, Writing, and Speaking and Listening have been designed to include the essential skills and knowledge that students need to apply to college and career tasks, such as research and media. Rather than having an additional set of standards that would largely duplicate those already in Reading, Writing, and Speaking and Listening, the document includes the Research and Media applications that draw upon standards already in those strands. This both reaffirms the centrality of the core processes of reading, writing, speaking, and listening and shows how those processes can be combined and extended to describe key communicative acts in the classroom and workplace.

In the Research and Media applications, specific Reading, Writing, and Speaking and Listening standards are identified with a letter corresponding to the relevant strand (R for Reading, W for Writing, and S&L for Speaking and Listening) and a number or letter corresponding to the statement within that strand. For example, R-14 refers to the fourteenth statement in the Standards for Student Performance in Reading, and W-A refers to the first statement of the Standards for Range and Content in Writing.

College and Career Readiness Standards for Reading, Writing, and Speaking and Listening

Draft for Review and Comment

September 21, 2009

College and Career Readiness Standards for Reading, Writing, and Speaking and Listening

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Core Standards for Reading, Writing, and Speaking and Listening

The Core Standards identify essential college- and career-ready skills and knowledge in reading, writing, and speaking and listening across the disciplines. While the English language arts classroom has often been seen as the proper site for literacy instruction, this document acknowledges that the responsibility for teaching such skills must also extend to the other content areas. Teachers in the social and natural sciences, the humanities, and mathematics need to use their content area expertise to help students acquire the discipline-specific skills necessary to comprehend challenging texts and develop deep knowledge in those fields. At the same time, English language arts teachers not only must engage their students in a rich array of literature but also must help develop their students' ability to read complex works of nonfiction independently.

What is taught is just as important as how it is taught; the Core Standards should be accompanied by a comprehensive, content-rich curriculum. While this document defines the outcomes all students need to reach to be college and career ready, many important decisions about curriculum will necessarily be left to states, districts, schools, teachers, professional organizations, and parents. For example, while the standards require that students read texts of sufficient complexity, quality, and range, this document does not contain a required reading list. If states and districts choose to develop one, they should look at the Reading exemplars provided here to get a sense of the level of complexity students must be able to handle independently when they read. Educators can also model their efforts on reading lists from around the nation and the world as long as the texts ultimately included meet the range and content standards in this document.

Standards today must ready students for competition and collaboration in a global, media-saturated environment. Colleges and universities have become international meetinghouses where people from across the globe learn with and from one another. At the same time, business today is truly a worldwide enterprise. Media-related technology helps shape what goes on in both college and the workplace; indeed, it has in some important ways reshaped the very nature of communication. Students who meet the Core Standards will have the reading, writing, speaking, and listening skills to flourish in the diverse, rapidly changing environments of college and careers.

Although reading, writing, and speaking and listening are articulated separately in the standards that follow, these divisions are made for the sake of clarity and manageability. In reality, the processes of communication are tightly interrelated and often reciprocal. The act of reading can no more be separated from the written word than the act of listening can be from the spoken word. When reading, students demonstrate their comprehension most commonly through a spoken or written interpretation of the text. As students solve problems, share insights, and build the knowledge they need for college and career success, they draw simultaneously on their capacities to read, write, speak, and listen.

Student Practices in Reading, Writing, and Speaking and Listening

The following practices in reading, writing, and speaking and listening undergird and help unify the rest of the standards document. They are the "premises"—broad statements about the nature of college and career readiness in reading, writing, and speaking and listening—that underlie the individual standards statements and cut across the various sections of the document. Every idea introduced here is subsequently represented in one or more places within the larger document.

* * *

Students who are college and career ready exhibit the following capacities in their reading, writing, and speaking and listening:

1. *They demonstrate independence as readers, writers, speakers, and listeners.*

   Students can, without significant scaffolding or support, comprehend and evaluate complex text across a range of types and disciplines, and they can construct effective arguments and clearly convey intricate or multifaceted information. Likewise, students are independently able to discern a speaker's key points as well as ask questions and articulate their own ideas.

2. *They build strong content knowledge.*

   Students build a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They demonstrate their ability to become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and the specific in-depth expertise needed to comprehend subject matter and solve problems in different fields. They refine their knowledge and share it through substantive writing and speaking.

3. *They respond to the varying demands of audience, task, purpose, and discipline.*

   Students consider their reading, writing, and speaking and listening in relation to the contextual factors of audience, task, purpose, and discipline. They appreciate nuances, such as how the composition and familiarity of the audience should affect tone. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in the natural sciences).

4. *They comprehend as well as critique.*

   Students are engaged and open-minded—but skeptical—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and assess the veracity of claims.

5. *They privilege evidence.*

   Students cite specific textual evidence when offering an oral or written interpretation of a piece of writing. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

6. *They care about precision.*

   Students are mindful of the impact of specific words and details, and they consider what would be achieved by different choices. Students pay especially close attention when precision matters most, such as in the case of reviewing significant data, making important distinctions, or analyzing a key moment in the action of a play or novel.

7. *They craft and look for structure.*

   Students attend to structure when organizing their own writing and speaking as well as when seeking to understand the work of others. They understand and make use of the ways of presenting information typical of different disciplines. They observe, for example, how authors of literary works craft the structure to unfold events and depict the setting.

8. *They use technology strategically and capably.*

   Students employ technology thoughtfully to enhance their reading, writing, speaking, and listening. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

Introductory Evidence Statement for Reading, Writing, and Speaking and Listening Standards

To develop college- and career-ready standards for Reading, Writing, and Speaking and Listening that are rigorous, relevant, and internationally benchmarked, the work group consulted evidence from a wide array of sources. These included standards documents from high-performing states and nations; student performance data (including assessment scores and college grades); academic research; frameworks for assessments, such as NAEP; and results of surveys of postsecondary instructors and employers regarding what is most important for college and career readiness.

The evidence strongly suggests that similar reading, writing, speaking, and listening skills are necessary for success in both college and the workplace. A review of the standards of high-performing nations also suggests that many of these skills are already required in secondary schools internationally. The work group has endeavored to articulate these skills in the Core Standards, focusing educators, students, parents, and resources on what matters most.

Given that a set of standards cannot be simplistically "derived" from any body of evidence, the work group sometimes relied on reasoned judgment to interpret where the evidence was most compelling. For example, there is not a consensus among college faculty about the need for incoming students to be able to comprehend graphs, charts, and tables and to integrate information in these data displays with the information in the accompanying text. Although some evidence suggests that this skill is critical in the workplace and in some entry-level courses, college faculties from the various disciplines disagree on its value (with science and economics faculty rating it more highly than English and humanities professors do). The work group ultimately included a standard on the integration of text and data because the preponderance of the evidence suggests the skill's importance in meeting the demands of the twenty-first-century workplace and some college classrooms.

In most cases, the evidence is clearer. In writing, for example, there is unequivocal value placed on the logical progression of ideas. The expectation that high school graduates will be able to produce writing that is logical and coherent is found throughout the standards of top-performing countries and states. This ability is also valued highly by college faculty and employers. In response to such clear evidence, the work group included Writing student performance standard #5: "Create a logical progression of ideas or events, and convey the relationships among them."

A bibliography of some of the sources the work group drew upon most is included at the end of this document. The reader should also refer to the Core Standards Web site (http://www.corestandards.org), which contains a list of standards linked to relevant sources of evidence.

Finally, while the standards reflect the best evidence available to date, the decisions the work group made are necessarily provisional. The core should be reexamined periodically as additional research on college and career readiness emerges. Indeed, this document may serve as an agenda for such research.

How to Read the Document

This document is divided into three main sections: strands, applications, and supporting materials.

Strands
There are three *strands*: Reading, Writing, and Speaking and Listening. Although each strand is presented discretely for ease of understanding, the document should be considered a coherent whole.

The three strands are each in turn divided into two sections: *Standards for Range and Content* and *Standards for Student Performance*.

Standards for Range and Content
The Standards for Range and Content in each strand describe the contexts in which college- and career-ready students must be able to read, write, speak, and listen. Rather than merely supplement or illustrate the numbered list of Standards for Student Performance, the Standards for Range and Content are themselves required and carry equal force.

Standards for Student Performance
The Standards for Student Performance in each strand enumerate the essential skills and understandings that students who are college and career ready in reading, writing, speaking, and listening must have no later than the end of high school.

Applications
The clearest examples of the integrated nature of communication are the *Applications of the Core* for Research and Media. The Core Standards for Reading, Writing, and Speaking and Listening have been designed to include the essential skills and knowledge that students need to apply to college and career tasks, such as research and media. Rather than having an additional set of standards that would largely duplicate those already in Reading, Writing, and Speaking and Listening, the document includes the Research and Media applications that draw upon standards already in those strands. This both reaffirms the centrality of the core processes of reading, writing, speaking, and listening and shows how those processes can be combined and extended to describe key communicative acts in the classroom and workplace.

In the Research and Media applications, specific Reading, Writing, and Speaking and Listening standards are identified with a letter or letters corresponding to the relevant strand (R for Reading, W for Writing, and S&L for Speaking and Listening) and a number or letter corresponding to the statement within that strand. For example, R-14 refers to the fourteenth statement in the Standards for Student

Performance in Reading, and W-A refers to the first statement of the Standards for Range and Content in Writing.

Supporting Materials: Reading and Writing Exemplars

Reading and Writing exemplars, and their accompanying annotations, are used to lend further specificity to the standards.

Reading Exemplars

The Reading exemplars, representing a range of subject areas, time periods, cultures, and formats, illustrate the level of text complexity students ready for college and careers must be able to handle on their own. The exemplars are mostly excerpts or representations of larger works. To be truly college and career ready, students must be able to handle full texts—poems, short stories, novels, technical manuals, research reports, and the like. Annotations accompanying the exemplars explain how each text meets the criterion of high text complexity. The annotations also provide brief performance examples that further clarify the meaning and application of the standards.

~~Writing Exemplars~~ - Coming in the next draft

~~The Writing exemplars are authentic samples of student writing created across the nation under a variety of conditions and for a variety of purposes and audiences. Annotations accompanying the exemplars indicate how these samples meet the Standards for Student Performance in Writing.~~

Core Standards for Reading Informational and Literary Texts

Standards for the Range and Content of Student Reading

A. Complexity: A crucial factor in readiness for college and careers is students' ability to comprehend complex texts independently. In college and careers, students will need to read texts characterized by demanding vocabulary, subtle relationships among ideas or characters, a nuanced rhetorical style and tone, and elaborate structures or formats. These challenging texts require the reader's close attention and often demand rereading in order to be fully understood.

B. Quality: The literary and informational texts chosen for study should be rich in content and in a variety of disciplines. All students should have access to and grapple with works of exceptional craft and thought both for the insights those works offer and as models for students' own thinking and writing. These texts should include classic works that have broad resonance and are alluded to and quoted often, such as influential political documents, foundational literary works, and seminal historical and scientific texts. Texts should also be selected from among the best contemporary fiction and nonfiction and from a diverse range of authors and perspectives.

C. Vocabulary: To be college and career ready, students must encounter and master a rich vocabulary. Complex texts often use challenging words, phrases, and terms that students typically do not encounter in their daily lives. Specific disciplines and careers have vocabularies of their own. Attentive reading of sophisticated works in a wide range of fields, combined with close attention to vocabulary, is essential to building comprehension and knowledge.

D. Range: Students must be able to read a variety of literature, informational texts, and multimedia sources in order to gain the knowledge base they need for college and career readiness.

   Literature: Literature enables students to access through imagination a wide range of experiences. By immersing themselves in literature, students enlarge their experiences and deepen their understanding of their own and other cultures. Careful reading of literature entails attentiveness to craft and details of design, which has broad value for students' work in college and career environments.

   Informational Text: Because most college and workplace reading is nonfiction, students need to hone their ability to acquire knowledge from informational texts. Workplace and discipline-specific reading will often require students to demonstrate persistence as they encounter a large amount of unfamiliar and often technical vocabulary and concepts. Students must demonstrate facility with the features of texts particular to a variety of disciplines, such as history, science, and mathematics.

   Multimedia Sources: Students must be able to integrate what they learn from reading text with what they learn from audio, video, and other digital media. Many of the same critical issues that students face when reading traditional printed texts will arise as they seek to comprehend multimedia, such as determining where the author has chosen to focus, evaluating evidence, and comparing different accounts of similar subjects.

E. Quantity*:* Students must have the capacity to handle independently the quantity of reading material, both in print and online, required in college and workforce training. Studies show that the amount of reading students face in high school is often far lower than that required for typical first-year college courses. Students need to be able to perform a close reading of a much higher volume of texts and to sort efficiently through large amounts of print and online information in search of specific facts or ideas.

Note: The essential role of independence in college and career readiness*:* The significant scaffolding that often accompanies reading in high school usually disappears in college and workforce training environments. Students must therefore have developed their ability to read texts of sufficient complexity, quality, and range on their own. To become independent, students must encounter unfamiliar texts presented without supporting materials.

Core Standards for Reading Informational and Literary Texts

Standards for Student Performance

1. Determine both what the text says explicitly and what can be inferred logically from the text.
2. Support or challenge assertions about the text by citing evidence in the text explicitly and accurately.
3. Discern the most important ideas, events, or information, and summarize them accurately and concisely.
4. Delineate the main ideas or themes in the text and the details that elaborate and support them.
5. Determine when, where, and why events unfold in the text, and explain how they relate to one another.
6. Analyze the traits, motivations, and thoughts of individuals in fiction and nonfiction based on how they are described, what they say and do, and how they interact.
7. Determine what is meant by words and phrases in context, including connotative meanings and figurative language.
8. Analyze how specific word choices shape the meaning and tone of the text.
9. Analyze how the text's organizational structure presents the argument, explanation, or narrative.
10. Analyze how specific details and larger portions of the text contribute to the meaning of the text.
11. Synthesize data, diagrams, maps, and other visual elements with words in the text to further comprehension.
12. Extract key information efficiently in print and online using text features and search techniques.
13. Ascertain the origin, credibility, and accuracy of print and online sources.
14. Evaluate the reasoning and rhetoric that support an argument or explanation, including assessing whether the evidence provided is relevant and sufficient.
15. Analyze how two or more texts with different styles, points of view, or arguments address similar topics or themes.
16. Draw upon relevant prior knowledge to enhance comprehension, and note when the text expands on or challenges that knowledge.
17. Apply knowledge and concepts gained through reading to build a more coherent understanding of a subject, inform reading of additional texts, and solve problems.
18. Demonstrate facility with the specific reading demands of texts drawn from different disciplines, including history, literature, science, and mathematics.

Note: These Standards for Student Performance, as is the case for every strand, must be demonstrated across the range and content from the preceding page. They are meant to apply to fiction and nonfiction. For example:

- "Determine when, where, and why events unfold" applies to plot and setting in literature as well as the sequence of a scientific procedure.
- "Analyze the traits, motivations, and thoughts of individuals" applies to studying characters in fiction and figures in historical texts.

Core Standards for Writing

Standards for the Range and Content of Student Writing

A. **Purpose:**

Make an Argument: While many high school students have experience presenting their opinions, they need to be able to make arguments supported by evidence in order to be ready for careers and college. Students must be able to frame the debate over a claim, present the reasoning and evidence for the argument, and acknowledge and address its limitations. In some cases, students will make arguments to gain entry to college or to obtain a job, laying out their qualifications or experience. In college, students might defend an interpretation of a work of literature or of history; in the workplace, employees might write to recommend a course of action.

Inform or Explain: In college and in workforce training, writing is a key means for students to show what they know and to share what they have seen. Writing to inform or explain often requires students to integrate complex information from multiple sources in a lucid fashion. Explanations can take the form of laying out facts about a new technology or documenting findings from historical research; well-crafted explanations often make fresh connections and express ideas creatively.

B. **Audience:** Students must adapt their writing so that it is appropriate to the audience by choosing words, information, structures, and formats that conform to the conventions of the discipline in which they are writing. The form and use of evidence in literary analysis, for example, are likely to be quite different from those in geology or business. Students must also be able to consider their audience's background knowledge and potential objections to an argument.

C. **Situation:**

On-demand Writing: Students must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline. College and career readiness requires that students be able to write effectively to a prompt on an exam or respond quickly yet thoughtfully to a supervisor's urgent request for information.

Writing over Time: Students must be able to revisit and make improvements to a piece of their writing over multiple drafts when circumstances encourage or require it. To improve writing through revision, students must be capable of distinguishing good changes from ones that would weaken the writing.

D. **Technology and Collaboration:** Technology offers students powerful tools for producing, editing, and distributing writing as well as for collaboration. Especially in the workplace, writers often use technology to produce documents and to provide feedback.

E. **Quantity:** The evidence is clear that, in order to become better writers, students must devote significant time to producing writing. Students must practice writing several analytical pieces each term if they are to achieve the deep analysis and interpretation of content expected for college and careers.

Note on narrative writing:

Narrative writing is an important mode of writing; it is also a component of making an argument and writing to inform or explain. Telling an interesting story effectively or providing an accurate account of a historical incident requires the skillful use of narrative techniques. Narrative writing requires that students present vivid, relevant details to situate events in a time and place and also craft a structure that lends a larger shape and significance to those details. As an easily grasped and widely used way to share information and ideas with others, narrative writing is a principal stepping-stone to writing forms directly relevant to college and career readiness.

Core Standards for Writing

Standards for Student Performance

1. Establish and refine a topic or thesis that addresses the specific task and audience.
2. Gather the information needed to build an argument, provide an explanation, or address a research question.
3. Sustain focus on a specific topic or argument.
4. Support and illustrate arguments and explanations with relevant details, examples, and evidence.
5. Create a logical progression of ideas or events, and convey the relationships among them.
6. Choose words and phrases to express ideas precisely and concisely.
7. Use varied sentence structures to engage the reader and achieve cohesion between sentences.
8. Develop and maintain a style and tone appropriate to the task, purpose, and audience.
9. Demonstrate command of the conventions of standard written English, including grammar, usage, and mechanics.
10. Represent and cite accurately the data, conclusions, and opinions of others, effectively incorporating them into one's own work while avoiding plagiarism.
11. Assess the quality of one's own writing, and, when necessary, strengthen it through revision.
12. Use technology as a tool to produce, edit, and distribute writing.

When **writing to inform or explain**, students must also do the following:

13. Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to provide an accurate picture of that information.
14. Convey complex information clearly and coherently to the audience through purposeful selection and organization of content.
15. Demonstrate understanding of content by reporting facts accurately and anticipating reader misconceptions.

When **writing arguments**, students must also do the following:

16. Establish a substantive claim, distinguishing it from alternate or opposing claims.
17. Link claims and evidence with clear reasons, and ensure that the evidence is relevant and sufficient to support the claims.
18. Acknowledge competing arguments or information, defending or qualifying the initial claim as appropriate.

Note: "The conventions of standard written English" encompass a range of commonly accepted language practices designed to make writing clear and widely understood. When formal writing contains errors in grammar, usage, and mechanics, its meaning is obscured, its message is too easily dismissed, and its author is often judged negatively. Proper sentence structure, correct verb formation, careful use of verb tense, clear subject-verb and pronoun-antecedent agreement, conventional usage, and appropriate punctuation are of particular importance to formal writing.

Core Standards for Speaking and Listening

Standards for the Range and Content of Student Speaking and Listening

A. **Group and One-to-One Situations:** Students are expected to be able to speak and listen effectively in both groups and one-to-one. Success in credit-bearing college coursework, whether in the humanities, mathematics, or the sciences, depends heavily on being able to take in and respond to the concepts and information conveyed in lectures and class discussions. Success in the workplace is similarly dependent on listening attentively to colleagues and customers and expressing ideas clearly and persuasively.

   These speaking and listening skills may need to be applied differently in different settings. The immediate communication between two people might be replaced by formal turn taking in large-group discussions. When working in classroom or workplace teams, students should be able to ask questions that initiate thoughtful discussions, gain the floor in respectful ways, and build on the contributions of others to complete tasks or reach consensus.

B. **Varied Disciplinary Content:** Students must adapt their speaking and listening to a range of disciplines to communicate effectively. Each academic discipline and industry has its own vocabulary and conventions; for instance, evidence is handled and discussed differently in literary analysis than in history or medicine or the sciences. College- and career-ready students must develop a foundation of disciplinary knowledge and conventions in order not only to comprehend the complexity of information and ideas but also to present and explain them.

C. **Multimedia Comprehension:** New technologies expand the role that speaking and listening skills will play in acquiring and sharing knowledge. Students will need to view and listen to diverse media to gain knowledge and also must integrate this information with what they learn through reading text online as well as in print. When speaking, students can draw on media to illustrate their points, make data and evidence vivid, and engage their audience. Multimedia accelerates the speed at which connections between reading, writing, speaking, and listening can be made, requiring students to be ready to use these skills nearly simultaneously.

Core Standards for Speaking and Listening

Standards for Student Performance

1. Select and use a format, organization, and style appropriate to the topic, purpose, and audience.
2. Present information, findings, and supporting evidence clearly and concisely.
3. Make strategic use of multimedia elements and visual displays of data to gain audience attention and enhance understanding.
4. Demonstrate command of formal Standard English when appropriate to task and audience.
5. Listen to complex information, and discern the main ideas, the significant details, and the relationships among them.
6. Follow the progression of the speaker's message, and evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric.
7. Ask relevant questions to clarify points and challenge ideas.
8. Respond constructively to advance a discussion and build on the input of others.

Note: "Style appropriate to the topic, purpose, and audience" includes word choice specific to the demands of the discipline as well as delivery techniques such as gestures and eye contact that contribute to effective message delivery.

"Evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric" includes distinguishing facts from opinions and determining whether the speaker is biased and evidence has been distorted.

Application of the Core: Research

The Core Standards for Reading, Writing, and Speaking and Listening have been designed to include the essential skills and knowledge that students need to apply to college and career tasks such as research. This section shows how standards in the core incorporate the skills of research.

To be college and career ready, students must engage in research and present their findings in writing and orally, in print and online. The ability to conduct research independently and effectively plays a fundamental role in gaining knowledge and insight in college and the workplace.

Research as described here is not limited to the formal, extended research paper nor simply to gathering information from books; rather, research encompasses a flexible yet systematic approach to resolving questions and investigating issues through the careful collection, analysis, synthesis, and presentation of information from a wide range of print and digital sources, such as historical archives and online interviews. With well-developed research skills, students have the tools to engage in sustained inquiry as well as the sort of short, focused research projects that typify many assignments in college and the workplace.

Research in the digital age offers new possibilities as well as new or heightened challenges. While the Internet provides ready access to unprecedented amounts of primary and secondary source material (such as oral histories, historical documents, maps, and scientific reports), students sorting through this wealth of data must be skilled at and vigilant in determining the origin and credibility of these sources.

The following Core Standards pertain to elements of the research process and particular research skills required for college and career readiness:

Formulate research questions:
- Establish and refine a topic or thesis that addresses the specific task and audience. (W-1)
- Establish a substantive claim, distinguishing it from alternate or opposing claims. (W-16)

Gather and evaluate relevant information from a range of sources:
- Gather the information needed to build an argument, provide an explanation or address a research question. (W-2)
- Extract key information efficiently in print and online using text features and search techniques. (R-12)
- Ascertain the origin, credibility, and accuracy of print and online sources. (R-13)
- Evaluate the reasoning and rhetoric that support an argument or explanation, including assessing whether the evidence provided is relevant and sufficient. (R-14)
- Follow the progression of the speaker's message and evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric. (S&L-6)

Analyze research sources:
- Delineate the main ideas or themes in the text and the details that elaborate and support them. (R-4)
- Listen to complex information and discern the main ideas, the significant details, and the relationships among them. (S&L-5)
- Discern the most important ideas, events, or information and summarize them accurately and concisely. (R-3)
- Synthesize data, diagrams, maps, and other visual elements with words in the text to further comprehension. (R-11)
- Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to provide an accurate picture of that information. (W-13)
- Analyze how two or more texts with different styles, points of view, or arguments address similar topics or themes. (R-15)
- Acknowledge competing arguments or information, defending or qualifying the initial claim as appropriate. (W-18)

Report findings:
- Link claims and evidence with clear reasons and ensure that the evidence is relevant and sufficient to support the claims. (W-17)
- Convey complex information clearly and coherently to the audience through purposeful selection and organization of the content. (W-14)
- Demonstrate understanding of the content by reporting the facts accurately and anticipating reader misconceptions. (W-15)
- Present information, findings, and supporting evidence, clearly and concisely. (S&L-2)
- Support and illustrate arguments and explanations with relevant details, examples, and evidence. (W-4)
- Represent and cite accurately the data, conclusions, and opinions of others, effectively incorporating them into one's own work while avoiding plagiarism. (W-10)

Application of the Core: Media

The Core Standards for Reading, Writing, and Speaking and Listening have been designed to include the essential skills and knowledge that students need to apply to college and career tasks such as media analysis and creation. This section shows how standards in the core apply to media.

Rapidly evolving technologies are powerful tools—but only for those who have the skills to put them to work. As the capability of the technology grows, students' command of these skills must only increase.

At the core of media mastery are the same fundamental capacities as are required offline in traditional print forms: an ability to access, understand, and evaluate complex materials and messages and to produce clear, effective communications. Media mastery does, however, call upon students to apply these core skills in new ways and contexts. Media enable students to communicate quickly with a large, often unknown, and broadly diverse audience. Whereas in the past, students may have had days or weeks to digest new information and formulate a response, the online environment pushes students to exercise judgment and present their responses in a matter of minutes.

Speed is not the only new factor. In the electronic world, reading, writing, speaking, and listening are uniquely intertwined. Multimedia forms force students to engage with constantly changing combinations of elements, such as graphics, images, hyperlinks, and embedded video and audio. The technology itself is changing quickly, creating new urgency for adaptation and flexibility on the part of students.

The following Core Standards describe the particular reading, writing, speaking, and listening skills that students will need in order to use media effectively in college and careers:

Standards for Range and Content drawn from each strand

Multimedia Sources: Students must be able to integrate what they learn from reading text with what they learn from audio, video, and other digital media. Many of the same critical issues that students face when reading traditional printed texts will arise as they seek to comprehend multimedia, such as determining where the author has chosen to focus, evaluating evidence, and comparing different accounts of similar subjects. [R-D]

Technology and Collaboration: Technology offers students powerful tools for producing, editing, and distributing writing as well as for collaboration. Especially in the workplace, writers often use technology to produce documents and to provide feedback. [W-D]

Multimedia Comprehension: New technologies expand the role that speaking and listening skills will play in acquiring and sharing knowledge. Students will need to view and listen to diverse media to gain knowledge and integrate this information with what they learn through reading text online as well as in print. When speaking, students can draw on media to illustrate their points, make data and evidence vivid, and engage their audiences. Multimedia accelerates the speed at which connections between reading, writing, and speaking and listening can be made, requiring students to be ready to use these skills nearly simultaneously. [S&L-C]

Standards for Student Performance drawn from each strand

Gather information from a wide array of electronic sources and multimedia:

- Extract key information efficiently in print and online using text features and search techniques. (R-12)
- Synthesize data, diagrams, maps, and other visual elements with words in the text to further comprehension. (R-11)
- Listen to complex information and discern the main ideas, the significant details, and the relationships among them. (S&L-5)

Evaluate information from digital media:

- Ascertain the origin, credibility, and accuracy of print and online sources. (R-13)
- Evaluate the reasoning and rhetoric that support an argument or explanation, including assessing whether the evidence provided is relevant and sufficient. (R-14)
- Follow the progression of the speaker's message and evaluate the speaker's point of view, reasoning, and use of evidence and rhetoric. (S&L-6)

Create and distribute media communications:

- Use technology as a tool to produce, edit, and distribute writing. (W-12)
- Synthesize information from multiple relevant sources, including graphics and quantitative information when appropriate, to provide an accurate picture of that information. (W-13)
- Make strategic use of multimedia elements and visual displays of data to gain audience attention and enhance understanding. (S&L-3)

Illustrative Texts

Exemplars of Reading Text Complexity

As described in the Standards for the Range and Content of Student Reading, college- and career-ready students must be able to read texts of sufficient complexity on their own. Studies show that many students who are unable to read sufficiently challenging texts independently by the end of high school struggle with the reading demands of college; many twenty-first-century careers likewise demand that people be able to obtain, search through, and comprehend large amounts of often technical information.

To develop that ability, students should engage with high-quality texts that provide strong models of thinking and writing, that challenge them intellectually, and that introduce them to rich content, sophisticated vocabulary, and examples of exceptional craft. The reading students do should be broad and deep, allowing them to extend their knowledge of particular subjects as well as learn about the features of texts written for different disciplines, audiences, and purposes. While no sampling can do justice to the numerous ways in which different authors craft complex prose, as a collection the exemplar texts below illustrate the level of complexity that college- and career-ready students should be able to handle independently by the end of high school. Texts in translation have not been included in this draft but will be part of future drafts.

How Text Complexity was Determined

In addition to surveys of required reading in twelfth grade and the first year of college as well as consultations with experts, two leading measurement systems were used to help make the selections below. The first system—a methodology described by Jeanne Chall and her coauthors in *The Qualitative Assessment of Text Difficulty*—employs trained raters to measure the sophistication of vocabulary, density of ideas, and syntactic complexity in a text as well as the general and subject-specific knowledge and the level of reasoning required for understanding it. The second system, Coh-Metrix, incorporates into its computer-based analysis more than sixty specific indices of syntax, semantics, readability, and cohesion to assess text complexity. Central to its assessment are measures of text cohesiveness, which is the degree to which the text uses explicit markers to link ideas. By analyzing the degree to which those links are missing in a text—and therefore the degree to which a reader must make inferences to connect ideas—this measure gauges a key factor in the comprehension demand of a text.

The two methods described above have limitations. The complexity of poems (such as "O Captain! My Captain!") cannot be assessed by Coh-Metrix because poetry adheres to different rules of construction than does prose. Similarly, while individual stories in the sample *New York Times* front pages can be measured for complexity by Coh-Metrix, the method does not capture how the electronic environment enhances or detracts from readability. However, for those exemplar texts whose complexity could be measured by both systems, comparable results were yielded by Coh-Metrix and the Chall method.

Note: The samples of complex text are supplemented by brief performance examples that further clarify the meaning of the standards. These illustrate specifically the application of the performance standards to texts of sufficient complexity, quality, and range. Relevant standards are noted in brackets following each sample performance.

Notes on Illustrative Text #1

Pride and Prejudice by Jane Austen

Jane Austen's *Pride and Prejudice* is a sophisticated literary text featuring multiple plotlines, a style and word choice reflective of its time period and setting, and subtle relationships among characters; the excerpt here can only illustrate some of the complexities that readers of the full work will encounter. The novel's opening sentence—"It is a truth universally acknowledged, that a single man in possession of a good fortune, must be in want of a wife"—signals that today's readers will need to employ literary imagination and historical context to re-create for themselves a world largely in the past. The novel's style is elaborate, with many lengthy and, to the modern ear, formal-sounding sentences typical of the period during which the novel was written. While the dialogue is less formal than much of the surrounding text, words and phrases such as *let* (to mean "rent" or "lease") and *chaise and four* (referring to a type of carriage) mark the novel's setting. The excerpt suggests also the kind of close reading of the subtleties of character that readers must perform. The banter between Mr. and Mrs. Bennet reveals both affection and difference of opinion, and it offers clues to the mores of well-to-do English society in the early nineteenth century.

Sample performance aligned with the Core Standards

Students analyze the first impressions given of Mr. and Mrs. Bennet in the first chapter of *Pride and Prejudice* based on how the characters are described, what they say and do, and how they interact. Students compare these first impressions with their later understanding based on how the characters develop throughout the novel. [R-6]

Illustrative Text #1

from *Pride and Prejudice*

Chapter 1

It is a truth universally acknowledged, that a single man in possession of a good fortune, must be in want of a wife.

However little known the feelings or views of such a man may be on his first entering a neighbourhood, this truth is so well fixed in the minds of the surrounding families that he is considered as the rightful property of some one or other of their daughters.

"My dear Mr. Bennet," said his lady to him one day, "have you heard that Netherfield Park is let at last?"

Mr. Bennet replied that he had not.

"But it is," returned she; "for Mrs. Long has just been here, and she told me all about it."

Mr. Bennet made no answer.

"Do not you want to know who has taken it?" cried his wife impatiently.

"*You* want to tell me, and I have no objection to hearing it."

This was invitation enough.

"Why, my dear, you must know, Mrs. Long says that Netherfield is taken by a young man of large fortune from the north of England; that he came down on Monday in a chaise and four to see the place, and was so much delighted with it, that he agreed with Mr. Morris immediately; that he is to take possession before Michaelmas, and some of his servants are to be in the house by the end of next week."

"What is his name?"

"Bingley."

"Is he married or single?"

"Oh! single, my dear, to be sure! A single man of large fortune; four or five thousand a year. What a fine thing for our girls!"

"How so? how can it affect them?"

"My dear Mr. Bennet," replied his wife, "how can you be so tiresome! You must know that I am thinking of his marrying one of them."

"Is that his design in settling here?"

"Design! nonsense, how can you talk so! But it is very likely that he *may* fall in love with one of them, and therefore you must visit him as soon as he comes."

"I see no occasion for that. You and the girls may go, or you may send them by themselves, which perhaps will be still better, for as you are as handsome as any of them, Mr. Bingley might like you the best of the party."

"My dear, you flatter me. I certainly *have* had my share of beauty, but I do not pretend to be any thing extraordinary now. When a woman has five grown-up daughters she ought to give over thinking of her own beauty."

"In such cases a woman has not often much beauty to think of."

"But, my dear, you must indeed go and see Mr. Bingley when he comes into the neighbourhood."

"It is more than I engage for, I assure you."

"But consider your daughters. Only think what an establishment it would be for one of them. Sir William and Lady Lucas are determined to go, merely on that account, for in general, you know, they visit no new-comers. Indeed you must go, for it will be impossible for us to visit him if you do not."

"You are over-scrupulous surely. I dare say Mr. Bingley will be very glad to see you; and I will send a few lines by you to assure him of my hearty consent to his marrying whichever he chuses of the girls: though I must throw in a good word for my little Lizzy."

"I desire you will do no such thing. Lizzy is not a bit better than the others; and I am sure she is not half so handsome as Jane, nor half so good-humoured as Lydia. But you are always giving *her* the preference."

"They have none of them much to recommend them," replied he; "they are all silly and ignorant, like other girls; but Lizzy has something more of quickness than her sisters."

"Mr. Bennet, how can you abuse your own children in such a way! You take delight in vexing me. You have no compassion on my poor nerves."

"You mistake me, my dear. I have a high respect for your nerves. They are my old friends. I have heard you mention them with consideration these twenty years at least."

"Ah! you do not know what I suffer."

"But I hope you will get over it, and live to see many young men of four thousand a year come into the neighbourhood."

"It will be no use to us if twenty such should come, since you will not visit them."

"Depend upon it, my dear, that when there are twenty, I will visit them all."

Mr. Bennet was so odd a mixture of quick parts, sarcastic humour, reserve, and caprice, that the experience of three-and-twenty years had been insufficient to make his wife understand his character. *Her* mind was less difficult to develope. She was a woman of mean understanding, little information, and uncertain temper. When she was discontented she fancied herself nervous. The business of her life was to get her daughters married; its solace was visiting and news.

Notes on Illustrative Text #2

"O Captain! My Captain!" by Walt Whitman

Though poetry's complexity cannot be assessed by the measures of readability used for the prose exemplars, "O Captain! My Captain!" by Walt Whitman clearly has many of the features of complex texts listed in the Standards for the Range and Content of Student Reading. Modern readers must work to understand what would have been obvious to readers in 1865: "O Captain! My Captain!" is an extended-metaphor poem intended to convey Whitman's and the North's grief over the assassination of Abraham Lincoln so near the conclusion of hostilities in the Civil War. Every element in the poem stands for something else, with the captain representing Lincoln, the ship representing the Union (or the "ship of state"), the voyage representing the war, and so on. Historical context, along with skill in reading literature, is thus particularly important to interpreting this text.

Sample performance aligned with the Core Standards

Students apply knowledge gained from reading the *New York Times* articles on Lincoln's assassination to their understanding of the poem "O Captain! My Captain!" Specifically, students draw on the description of the crowd's response to the attack on Lincoln to inform their understanding of Whitman's poem. [R-17]

Illustrative Text #2

"O Captain! My Captain!" by Walt Whitman

O Captain! my Captain! our fearful trip is done,
The ship has weather'd every rack, the prize we sought
is won,
The port is near, the bells I hear, the people all exulting,
While follow eyes the steady keel, the vessel grim and daring,
But O heart! heart! heart!
O the bleeding drops of red,
Where on the deck my Captain lies,
Fallen cold and dead.

O Captain! my Captain! rise up and hear the bells;
Rise up—for you the flag is flung—for you the bugle trills,
For you bouquets and ribbon'd wreaths—for you the shores
a-crowding,
For you they call, the swaying mass, their eager faces turning,
Here, Captain! dear father!
This arm beneath your head;
It is some dream that on the deck
You've fallen cold and dead.

My Captain does not answer, his lips are pale and still
My father does not feel my arm, he has no pulse nor will,
The ship is anchor'd safe and sound, its voyage closed
and done,
From fearful trip, the victor ship comes in with object won;
Exult, O shores, and ring O bells!
But I with mournful tread
Walk the deck my Captain lies,
Fallen cold and dead.

Notes on Illustrative Text #3

The front page of the *New York Times,* April 15, 1865

The challenge posed to a modern reader by the front page of the *New York Times* on April 15, 1865, is significant in terms of format, timeliness, and point of view. Unlike the graphically heavy front page of modern newspapers, this 1865 *New York Times* front page is mostly uninterrupted columns of text. The reader is obviously expected to proceed from top to bottom and left to right across the page, but little other guidance is provided. Because the assassination of Lincoln was still "breaking news" as this edition of the *Times* would have gone to press, some details of the event would have not yet been known; readers will have to sort out what they know about the assassination from what the people reading the paper on that Saturday morning would just have been learning. Three accounts of the events rather than one are provided here, and the sourcing and tone vary greatly. Certain details found in one place are contradicted in another: the "Detail of the Occurrence," for example, suggests that Lincoln may not have been mortally wounded, but the main headline in the top left-hand corner of the page states "No Hopes Entertained of His Recovery." While the first two accounts aim at a certain objectivity, the third begins with a flourish that may surprise readers more used to a restrained style of journalism: "A stroke from Heaven laying the whole of the city in instant ruin could not have startled us as did the word that broke from Ford's Theatre a half hour ago that the President had been shot."

Sample performance aligned with the Core Standards

Students analyze how the three different accounts on the front page portray Lincoln's assassination, including which details are similar or different. [R-15]

Illustrative Text #3: The front page of the *New York Times*, April 15, 1865

http://timesmachine.nytimes.com/browser/1865/04/15/P1

The New-York Times.

NEW-YORK, SATURDAY, APRIL 15, 1865.

PRICE FOUR CENTS

AWFUL EVENT.

President Lincoln Shot by an Assassin.

The Deed Done at Ford's Theatre Last Night.

THE ACT OF A DESPERATE REBEL

The President Still Alive at Last Accounts.

No Hopes Entertained of His Recovery.

Attempted Assassination of Secretary Seward.

EUROPEAN NEWS.

DETAILS OF THE DREADFUL TRAGEDY.
[OFFICIAL.]
War Department, Washington, April 15- 1:30 A.M.
Maj.-Gen. Dix:
This evening at about 9:30 P.M., at Ford's Theater, the President, while sitting in his private box with Mrs. Lincoln, Mrs. Harris, and Major Rathburn, was shot by an assassin, who suddenly entered the box and approached behind the President.
The assassin then leaped upon the stage, brandishing a large dagger or knife, and made his escape in the rear of the theater.

ANOTHER ACCOUNT.
Special Dispatch to the New-York Times.
Washington, Friday, April 14,
11:15 P.M.
A stroke from Heaven laying the whole of the city in instant ruin could not have startled us as did the word that broke from Ford's Theater a half hour ago that the President had been shot. It flew everywhere in five minutes, and set five thousand people in swift and excited motion on the instant.

DETAIL OF THE OCCURRENCE.
Washington, Friday, April 14- 12:30 A.M.
The President was shot in a theater tonight, and is, perhaps, mortally wounded. Secretary Seward was also assassinated.
SECOND DISPATCH.
Washington, Friday, April 14. President Lincoln and wife, with other friends, this evening visited Ford's Theater for the purpose of witnessing the performance of the "American Cousin."

Notes on Illustrative Text #4

The Declaration of Independence

The Declaration of Independence represents the kind of rich primary source material students should be able to read on their own by the end of high school. Though some of the lines ("We hold these truths . . .") are familiar to most American readers, the case against Great Britain that the Declaration lays out, expressed in elevated, sometimes archaic language (*unalienable, hath, usurpations*), requires careful examination to follow in its particulars. The beginning of the document, excerpted here, poses a reading challenge partly because of its philosophical abstractness. The first three sentences, although formally divided, are one continuous list of propositions ("truths") about the nature of government and the rights of the people. Further complicating the reading is that there is little explicit cohesion between sentences—links supplied by words and phrases such as "for example," "moreover," or "in addition"—to help readers understand the relationship between the ideas being expressed.

Sample performance aligned with the Core Standards

Students compare the argument that the Declaration makes justifying revolution to Martin Luther King, Jr.'s defense of civil disobedience in *Letter from Birmingham Jail*. [R-15]

Illustrative Text #4

from The Declaration of Independence

When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. —That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, —That whenever any Form of Government becomes destructive to these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new guards for their future security. —Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

Notes on Illustrative Text #5

Letter from Birmingham Jail by Martin Luther King, Jr.

Martin Luther King, Jr.'s, *Letter from Birmingham Jail* presents many challenges to the reader in terms of its format, purpose, tone, use of allusions, and language. Apart from letters to the editor (most of which are relatively short), public letters such as King's are uncommon today. The purpose of the text may also be confusing: King is ostensibly addressing his "Fellow Clergymen," but skilled readers will reasonably infer that King's message is intended for a broader audience. Though the tone of the text is measured, King's passion for his cause comes through. The author frequently points outside the *Letter* itself through allusions to other texts, including the Hebrew and Christian scriptures. Moreover, King uses sophisticated vocabulary (*cognizant, mutuality, provincial, gainsaying*) and figurative language (*garment of destiny*) throughout his text. However, the piece is both coherent in that its sequence is signaled ("While confined here . . . But more basically . . . Moreover, I am cognizant . . .") and cohesive in that its clauses and sentences are logically linked for the reader ("Just as the prophets . . . and just as the Apostle Paul . . . so am I compelled . . .").

Sample performance aligned with the Core Standards
Students evaluate the reasoning and rhetoric of the three very different arguments King makes to defend his being in Birmingham. Students assess the different kinds of evidence he uses to support each argument. [R-14]

Illustrative Text #5

from *Letter from Birmingham Jail**

My Dear Fellow Clergymen:

While confined here in the Birmingham city jail, I came across your recent statement calling my present activities "unwise and untimely." Seldom do I pause to answer criticism of my work and ideas. If I sought to answer all the criticisms that cross my desk, my secretaries would have little time for anything other than such correspondence in the course of the day, and I would have no time for constructive work. But since I feel that you are men of genuine good will and that your criticisms are sincerely set forth, I want to try to answer your statements in what I hope will be patient and reasonable terms.

I think I should indicate why I am here In Birmingham, since you have been influenced by the view which argues against "outsiders coming in." I have the honor of serving as president of the Southern Christian Leadership Conference, an organization operating in every southern state, with headquarters in Atlanta, Georgia. We have some eighty-five affiliated organizations across the South, and one of them is the Alabama Christian Movement for Human Rights. Frequently we share staff, educational and financial resources with our affiliates. Several months ago the affiliate here in Birmingham asked us to be on call to engage in a nonviolent direct-action program if such were deemed necessary. We readily consented, and when the hour came we lived up to our promise. So I, along with several members of my staff, am here because I was invited here I am here because I have organizational ties here.

But more basically, I am in Birmingham because injustice is here. Just as the prophets of the eighth century B.C. left their villages and carried their "thus saith the Lord" far beyond the boundaries of their home towns, and just as the Apostle Paul left his village of Tarsus and carried the gospel of Jesus Christ to the far corners of the Greco-Roman world, so am I. compelled to carry the gospel of freedom beyond my own home town. Like Paul, I must constantly respond to the Macedonian call for aid.

Moreover, I am cognizant of the interrelatedness of all communities and states. I cannot sit idly by in Atlanta and not be concerned about what happens in Birmingham. Injustice anywhere is a threat to justice everywhere. We are caught in an inescapable network of mutuality, tied in a single garment of destiny. Whatever affects one directly, affects all indirectly. Never again can we afford to live with the narrow, provincial "outside agitator" idea. Anyone who lives inside the United States can never be considered an outsider anywhere within its bounds.

*As reprinted in *Why We Can't Wait* by King, Jr., M. L. (2000). New York City: Signet Classics.

Notes on Illustrative Text #6

Toni Morrison's Nobel lecture, 1993

Toni Morrison's Nobel lecture, though originally delivered orally, can be read on the page as a complex work of analysis and criticism. Its structure, syntax, imagery, language, and density of ideas contribute to the challenge of studying it in this manner. As this excerpt shows, Morrison begins with a folktale. While the "once upon a time" opening may lead readers into thinking that the lecture will primarily be in narrative form, Morrison uses the tale mainly as a springboard for an abstract, allegorical discussion of language, writing, and those who have no voice in society. Morrison often employs sophisticated sentences that require patience and concentration to follow. Readers may recognize places where Morrison varies sentence patterns to change pace and rhythm—particularly important to the oral delivery of the text. The images Morrison creates are powerful and poetic, the diction is elevated and academic, and the word choice is metaphorical and unconventional: "Official language smitheryed to sanction ignorance and preserve privilege is a suit of armor polished to shocking glitter, a husk from which the knight departed long ago." The richness and abstractness of the ideas in the lecture mean that rereadings may be necessary to comprehend and evaluate the ideas fully.

Sample performance aligned with the Core Standards

Students determine what Morrison means when she compares language to "a bird in the hand," including the different connotations of this phrase that she develops throughout the lecture. Students also explore what Morrison means by saying that both the bird and language can be "dead or alive." [R-7]

Illustrative Text #6

from Toni Morrison's Nobel lecture, 1993

"Once upon a time there was an old woman. Blind but wise." Or was it an old man? A guru, perhaps. Or a griot soothing restless children. I have heard this story, or one exactly like it, in the lore of several cultures.

"Once upon a time there was an old woman. Blind. Wise."

In the version I know the woman is the daughter of slaves, black, American, and lives alone in a small house outside of town. Her reputation for wisdom is without peer and without question. Among her people she is both the law and its transgression. The honor she is paid and the awe in which she is held reach beyond her neighborhood to places far away; to the city where the intelligence of rural prophets is the source of much amusement.

One day the woman is visited by some young people who seem to be bent on disproving her clairvoyance and showing her up for the fraud they believe she is. Their plan is simple: they enter her house and ask the one question the answer to which rides solely on her difference from them, a difference they regard as a profound disability: her blindness. They stand before her, and one of them says, "Old woman, I hold in my hand a bird. Tell me whether it is living or dead."

She does not answer, and the question is repeated. "Is the bird I am holding living or dead?"

Still she doesn't answer. She is blind and cannot see her visitors, let alone what is in their hands. She does not know their color, gender or homeland. She only knows their motive.

The old woman's silence is so long, the young people have trouble holding their laughter.

Finally she speaks and her voice is soft but stern. "I don't know", she says. "I don't know whether the bird you are holding is dead or alive, but what I do know is that it is in your hands. It is in your hands."

Her answer can be taken to mean: if it is dead, you have either found it that way or you have killed it. If it is alive, you can still kill it. Whether it is to stay alive, it is your decision.

Whatever the case, it is your responsibility.

For parading their power and her helplessness, the young visitors are reprimanded, told they are responsible not only for the act of mockery but also for the small bundle of life sacrificed to achieve its aims. The blind woman shifts attention away from assertions of power to the instrument through which that power is exercised.

Speculation on what (other than its own frail body) that bird-in-the-hand might signify has always been attractive to me, but especially so now thinking, as I have been, about the work I do that has brought me to this company. So I choose to read the bird as language and the woman as a practiced writer. She is worried about how the language she dreams in, given to her at birth, is handled, put into service, even withheld from her for certain nefarious purposes. Being a writer she thinks of language partly as a system, partly as a living thing over which one has control, but mostly as agency—as an act with consequences. So the question the children put to her: "Is it living or dead?" is not unreal because she thinks of language as susceptible to death, erasure; certainly imperiled and salvageable only by an effort of the will. She believes that if the bird in the hands of her visitors is dead the custodians are responsible for the corpse. For her a dead language is not only one no longer spoken or written, it is unyielding language content to admire its own paralysis. Like statist language, censored and censoring. Ruthless in its policing duties, it has no desire or purpose other than maintaining the free range of its own narcotic narcissism, its own exclusivity and dominance. However moribund, it is not without effect for it actively thwarts the intellect, stalls conscience, suppresses human potential. Unreceptive to interrogation, it cannot form or tolerate new ideas, shape other thoughts, tell another story, fill baffling silences. Official language smitheryed to sanction ignorance and preserve privilege is a suit of armor polished to shocking glitter, a husk from which the knight departed long ago. Yet there it is: dumb, predatory, sentimental. Exciting reverence in schoolchildren, providing shelter for despots, summoning false memories of stability, harmony among the public.

Notes on Illustrative Text #7

Inquiry into Life, 12th edition, by Sylvia S. Mader

These excerpts, and the prominent college-level biology textbook from which they are drawn, represent some of the challenges presented by complex writing in natural science, including discipline-specific terms (*covalent bond, plasma membrane, neurotransmitter*), everyday language used in specialized ways (*shell, channel*), abbreviations (H^+, *AChE*), and chains of cause-effect relationships that together describe sometimes elaborate processes. Although the figures the author, Sylvia S. Mader, refers to in the text are not included with these excerpts, students reading the larger work will have to integrate words, illustrations, and diagrams to make full sense of the ideas and concepts she describes. For these reasons and others, comprehension may be difficult for readers who have not had experience independently reading similar kinds of text and who lack a knowledge base in the subject. The author does employ a number of cohesive features to help readers understand the terminology and to link ideas. She repeats content words to let readers follow the flow of ideas; she sets up contrastive situations to illustrate the ideas (within, for example, the first and the third paragraphs below); and she uses transitional links ("In some synapses . . . In other synapses . . .") to help readers construct meaning.

Sample performance aligned with the Core Standards

Students discern the most important information in the description of covalent bonding and provide an accurate summary of the concept. [R-3]

Illustrative Text #7

from *Inquiry into Life*, 12th edition

A **covalent bond** results when two atoms share electrons in such a way that each atom has an octet of electrons in the outer shell. In a hydrogen atom, the outer shell is complete when it contains two electrons. If hydrogen is in the presence of a strong electron acceptor, it gives up its electron to become a hydrogen ion (H^+). But if this is not possible, hydrogen can share with another atom and thereby have a completed outer shell. For example, one hydrogen atom will share with another hydrogen atom. Their two orbitals overlap, and the electrons are shared between them. Because they share the electron pair, each atom has a completed outer shell.

* * * * * * * *

The passage of salt (NaCl) across a plasma membrane is of primary importance to most cells. The chloride ion (Cl^-) usually crosses the plasma membrane because it is attracted by positively charged sodium ions (Na^+). First sodium ions are pumped across a membrane, and then chloride ions simply diffuse through channels that allow their passage.

As noted in Figure 4.2*a*, the genetic disorder cystic fibrosis results from a faulty chloride channel. Ordinarily, after chloride ions have passed though the membrane, sodium ions (Na^+) and water follow. In cystic fibrosis, Cl^- transport is reduced, and so is the flow of Na^+ and water.

* * * * * * * *

Once a neurotransmitter has been released into a synaptic cleft and has initiated a response, it is removed from the cleft. In some synapses, the postsynaptic membrane contains enzymes that rapidly inactivate the neurotransmitter. For example, the enzyme **acetylcholinesterase (AChE)** breaks down acetylcholine. In other synapses, the presynaptic membrane rapidly reabsorbs the neurotransmitter, possibly for repackaging in synaptic vesicles or for molecular breakdown. The short existence of neurotransmitters at a synapse prevents continuous stimulation (or inhibition) of postsynaptic membranes.

Notes on Illustrative Text #8

Sample business memo (ACT WorkKeys Reading for Information Test)

Though not a typical kind of reading in high school classrooms, the business communication, such as the one sampled here, is a form that career-ready students will need to be able to comprehend independently. This text, taken from ACT's WorkKeys Reading for Information Test, is challenging in large part because, like many such communications, it contains important, detailed information intended for a specialized audience. Structurally, the text offers little guidance on how it should be read. Potentially vital details appear throughout and are mingled with other details irrelevant to some readers (e.g., those without children). Even the paragraphing is somewhat inconsistent, especially between the first and second paragraphs. While the sentences are not particularly long and the language is not overly technical, the density of information and its lack of prioritization make this a complex text.

Sample performance aligned with the Core Standards

Students infer from the memo the conditions under which children who are under nineteen are not covered by the health plan. [R-1]

Illustrative Text #8

Sample business memo

WorkKeys Reading for Information Test has been reproduced with permission of ACT, Inc.

DETERMINING ELIGIBILITY FOR MEDICAL COVERAGE

All full-time employees of the company who work an average of at least 30 hours per week are eligible under this plan. Coverage begins on the first day of the month following the 30 days of active full-time employment. If employees enroll within 31 days of the date they are eligible, medical evidence of good health is not required. Temporary and part-time employees are not eligible. Employees are no longer eligible under this plan one month after the date they begin active duty in the armed forces of any country and continuing for the duration of their service.

If employees enroll their dependents within 31 days of the date they become eligible, medical evidence of good health is not required. If they do not, they will be required to submit evidence of good health for each dependent, at their expense, which is satisfactory to the company.

The following dependents are eligible under this plan: employees' spouses, employees' unmarried children under age 19, employees' unmarried dependent children under age 23 who are attending trade school, college, or university on a full-time basis, or employees' unmarried disabled children age 19 and over. Coverage ceases when spouses or children cease to be dependent upon employees for support. In the case of employees' spouses this is if they are legally separated or divorced. In the case of disabled children, this is when they are no longer disabled. Coverage will cease when dependents have served in the armed forces of any country for more than one month, or when maximum benefits have been paid.

Notes on Illustrative Text #9

FedViews, July 9, 2009, by Mary C. Daly
(The Federal Reserve Bank of San Francisco's Web site)

This text illustrates some of the difficulties posed by integrating information gained from words and graphics. This sort of challenge is common in writing designed to inform or explain, including writing in the workplace. The bullet point format used here means that the kind of explicit transitions between ideas typically found in prose are missing; readers will have to infer relationships between the points made by the author, Mary C. Daly, and synthesize the information into a coherent whole. Readers will furthermore have to analyze both the words and the graphics, integrate the information, and check to see whether each source of information supports the other. Daly also uses a great deal of specialized language; the terms *feedback loop, credit availability,* and *barriers to credit* all appear in just the first bullet point here.

Sample performance aligned with the Core Standards

Students synthesize information drawn from the text as well as the graphs in order to gain an overarching view of the economy on July 9, 2009. [R-11]

Illustrative Text #9

from FedViews, July 9, 2009

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Mary C. Daly, vice president and director of the Center for the Study of Innovation and Productivity at the Federal Reserve Bank of San Francisco, states her views on the current economy and the outlook.

- Financial markets are improving, and the crisis mode that has characterized the past year is subsiding. The adverse feedback loop, in which losses by banks and other lenders lead to tighter credit availability, which then leads to lower spending by households and businesses, has begun to slow. As such, investors' appetite for risk is returning, and some of the barriers to credit that have been constraining businesses and households are diminishing.

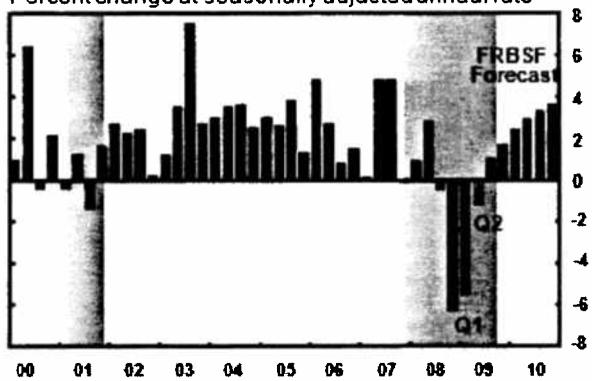
- Income from the federal fiscal stimulus, as well as some improvement in confidence, has helped stabilize consumer spending. Since consumer spending accounts for two-thirds of all economic activity, this is a key factor affecting our forecast of growth in the third quarter.

- The gradual nature of the recovery will put additional pressure on state and local budgets. Following a difficult 2009, especially in the West, most states began the 2010 fiscal year on July 1 with even larger budget gaps to solve.

- Still, many remain worried that large fiscal deficits will eventually be inflationary. However, a look at the empirical link between fiscal deficits and inflation in the United States shows no correlation between the two. Indeed, during the 1980s, when the United States was running large deficits, inflation was coming down.

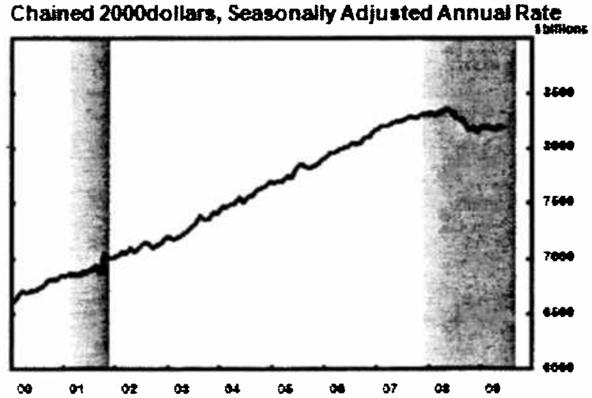
Modest recovery to begin in Q3

Real Gross Domestic Product (GDP)
Percent change at seasonally adjusted annual rate

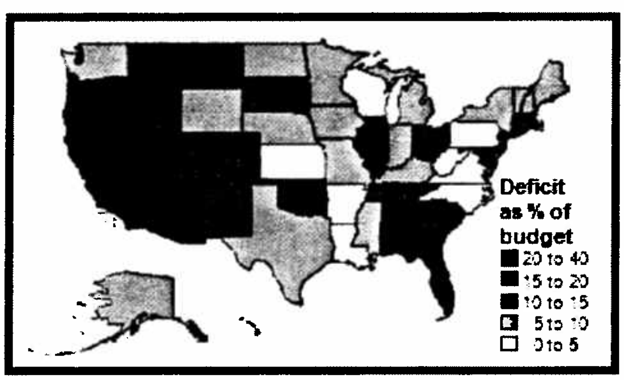


Consumers hanging on

Real Personal Consumption Expenditures
Chained 2000dollars, Seasonally Adjusted Annual Rate

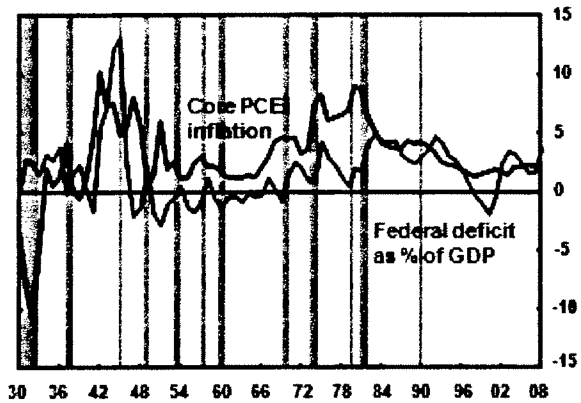


State budget gaps pervasive in 2009



No link between deficits and inflation

Fiscal deficits and core inflation



Notes on Illustrative Text #10

The front page of the *New York Times*, Web version, August 18, 2009, 9:03 a.m. ET

The challenge offered by this online text and others like it is very different from that offered by a complex continuous text in, say, the sciences. The brief passages are not conceptually difficult, the language is not technical or esoteric, and the sentences are not particularly complex. But these characteristics belie the complexity of the reading task. An online text of this kind requires readers to apply their print-reading skills in tandem with their knowledge of how to use online periodicals. The editors and designers have assigned levels of importance to individual stories and images, as measured by their size and position in the layout. The page itself uses words, numbers, icons, and other visual elements (e.g., line, color, and shape) to guide readers further. Headings in various colors direct readers to particular sections (OPINION, MARKETS, HEALTH), while links direct readers to particular stories ("Taliban Talks Are Key Issue in Afghan Vote"). Time markers ("3 minutes ago") help readers assess how new the information in a given story is. The text requires readers to make choices about which links to follow based on their understanding of how online text is typically structured and on a minimum of additional information (e.g., an icon of a camera, a drop-down menu in an ad).

Sample performance aligned with the Core Standards

Students select an article and use search terms and other features of the online text to research a specific aspect of the subject in more depth. [R-12]

Illustrative Text #10

The front page of the *New York Times*, Web version, August 18, 2009, 9:03 a.m. ET

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The New York Times

Tuesday, August 18, 2009 ... 9:03 AM ET

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BUSINESS
TECHNOLOGY
SPORTS
SCIENCE
HEALTH
OPINION
ARTS
STYLE
TRAVEL

Mental Stress Training Is Planned for U.S. Soldiers

A new Army program teaching mental resilience techniques is intended to prevent problems like post-traumatic stress disorder and depression.

- At War Blog: Notes From the Front Lines »

Post a Comment | Read (52)

Alternate Plan as Health Option Muddies Debate

The option of a co-op instead of a government health plan is so ill defined that no one knows exactly what it would look like or how effectively it would compete with commercial insurers.

Prescriptions Blog: Republicans Strike Back at Drug Makers

Inflation at a Low; New Home Construction Dips

Wholesale prices dropped sharply in July, and new home construction fell slightly, according to government reports.

Kim Dae-jung, 83, Ex-President of South Korea, Dies

Taliban Talks Are Key Issue in Afghan Vote

Above, supporters of Abdullah Abdullah, a candidate for the Afghan presidency. Whether and how to negotiate peace with the Taliban has become a top issue in the race.

- At War: Who Are the Taliban? | Post a Comment | Read (36)
- Afghanistan Hit by Attacks Ahead of Election

AT WAR »

Ask About the Afghan Elections

John Burns, The Times's chief foreign correspondent, is taking readers' questions.

Some N.Y. Lawmakers Take Pensions on Top of Pay

Some New York legislators "retire" one day and come back to work the next, a practice known as "double dipping."

One Person's Boondoggle, Another's Necessity

Some criticized stimulus projects are popular locally and underscore that boondoggle is in the eye of the beholder.

Post a Comment | Read (22)

Astor Trial Plods On, Trying the Patience of Many

The trial of Brooke Astor's son and one of her lawyers for fraud has lasted longer than anyone could have expected.

MORE NEWS

OPINION »

OP-ED

Afghanistan Votes

Four Afghans report voters' moods in their communities.

- Op-Art: Saturated Sharing

SCIENCE TIMES »

Tests Begin on Drugs That May Slow Aging

The drugs' development represents a new optimism that aging is not immutable.

Post a Comment

HEALTH »

A Transplant Race

Stretching the limits to save a teenager with a deadly and rare disease.

- Editorial: Climate and Security | Comments
- Editorial: Executive Pay
- Herbert: This Is Reform?
- Editorial Notebook: Hawaii
- Room for Debate: The Next Olympic Sport?

MARKETS »

Britain	Germany	France
FTSE 100	DAX	CAC 40

GET QUOTES — My Portfolios »

REAL ESTATE AUTOS JOBS ALL CLASSIFIEDS

Sample of Works Consulted

I. College Readiness

A. Achieve, Inc. (2004). The American Diploma Project, Ready or Not: Creating a High School Diploma that Counts. Washington, DC: Achieve, Inc. (PDF)

B. Achieve, Inc. (2008) Out of Many, One: Towards Rigorous Common Core Standards from the Ground Up. Washington, DC: Achieve, Inc. (PDF)

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B. ACT. (2007). *Aligning Postsecondary Expectations and High School Practice: The Gap Defined (Policy Implications of the ACT National Curriculum Survey Results 6005-6006).* Iowa City, IA: ACT. (PDF)

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E. Casner-Lotto, J., Rosenblum, E., and Wright, M., (2009). The *Ill-Prepared Workforce: Exploring the Challenges of Employer-Provided Workforce Readiness Training.* The Conference Board.

F. Florida American Diploma Project Survey Results. (Achieve).

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H. Missouri *Career Prep Certificate Program Planning Guide*, Missouri Department of Elementary and Secondary Education. (PDF)

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K. Partnership for 21st Century Skills. (2009). *Framework for 61st Century Learning*. Tucson, AZ.

L. Qualifications and Curriculum Authority. (2007). *Functional Skills and Standards.* (PDF)

M. The Conference Board. (2008). *New Graduates' Workforce*

Readiness.

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A. **Alberta, Canada:** English *L*anguage *A*rts *C*urriculum *O*utcomes, 2003. (PDF)

B. **British Columbia, Canada:** English *L*anguage *A*rts *I*ntegrated Resource *P*ackage, *P*rescribed *L*earning *O*utcomes, 2007. (PDF)

C. **British Columbia:** English Literature 12, Integrated Resource Package, 2003. (PDF)

D. **England:** English *P*rogramme of Study for *K*ey stage *4*, 2007. (PDF)

E. **Finland:** *N*ational *C*ore *C*urriculum for *U*pper Secondary Schools for Mother Tongue and Literature, Finnish as the mother tongue, 2003. (PDF)

F. **Hong Kong:** English *L*anguage *C*urriculum and *A*ssessment *G*uide, 2007. (PDF)

G. **Hong Kong:** Hong Kong Examinations and Assessment Authority, 2007 HKCEE English Language, Recommended Texts for the School-Based Assessment Component. (PDF)

H. **Ireland:** *L*eaving *C*ertificate/English Syllabus for *H*igher *L*evel and *O*rdinary *L*evel. (PDF)

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K. **New South Wales**: 2009-2012 HSC, Prescribed Area of Study, Electives and Texts. (PDF)

L. **Ontario, Canada:** The *O*ntario *C*urriculum, English, 2007. (PDF)

M. **Singapore:** English *L*anguage Syllabus *6001*. (PDF)

N. **Singapore:** O-Level Literature in English (Syllabus 2015), Prescribed Texts for 2009. (PDF)

O. **Victoria, Australia:** *V*ictorian *C*ertificate of Education Study *D*esign*:* English/English as a Second *L*anguage, 2006. (PDF)

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Exhibit C-3

College and Career Readiness Standards for Mathematics

Draft for Review and Comment

September 21, 2009

Contents 1

Introduction

The *College and Career Readiness Standards for Mathematics* consist of three interconnected parts: a Standard for Mathematical Practice, ten Standards for Mathematical Content, and a set of Example Tasks.

The Standard for Mathematical Practice has six Core Practices that describe the way proficient students approach mathematics. Proficient students attend to precision, construct viable arguments, make sense of complex problems and persevere in solving them, look for hidden structure, note regularity in repeated reasoning, and use technology intelligently. This approach to mathematics is an essential part of being ready for college and career.

The Standards for Mathematical Content form the backbone of this document. Each of these ten standards consists of Core Concepts, Core Skills, and a description of the student's Coherent Understanding. Students who encounter the subject with a focus on coherence will be better able to learn more mathematics at a deeper level and be better able to access and apply the mathematics they know. The ten Standards for Mathematical Content pull together topics previously studied and look ahead toward topics in further coursework and training programs.

The Standards for Mathematical Content are designed to draw greater attention to powerful organizing principles in mathematics, such as functional relationships or the laws of arithmetic. They also allow important distinctions to be made more clearly, such as that between Expressions and Equations. And they surface the deep connections that often underlie mathematical coherence, such as the blending of algebra with geometry represented by Coordinates. These ten are not categories or buckets of topics to cover; they are standards. They describe the coherence students need and deserve as they go forward to their mathematical futures.

The third component of the *College and Career Readiness Standards for Mathematics* is a Web-based collection of Example Tasks that exemplifies the variety of performances required. High standards demand that students *use* their knowledge, skills and good practices to solve problems from a variety of contexts, both within mathematics and from the world outside. Example Tasks exemplify the range and variety of use that is expected. Teachers and designers of curriculum and assessment will find in the collection of examples a guide to what these standards mean. Over time, the collection of tasks will grow.

Together, these three components establish an evidence-based standard for college and career readiness. The *College and Career Readiness Standards for Mathematics* have been created with attention to the expectations of the highest achieving countries. They have focus and depth, emphasizing the understanding of and connections among topics that are most important for success regardless of a student's pathway after reaching these standards.

A primary goal of developing these standards is to enable students to achieve *mathematical proficiency* (see sidebar). Students are expected to understand the knowledge described in the Core Concepts and in the Coherent Understandings at a depth that enables them to reason with that knowledge—to analyze, interpret and evaluate mathematical problems, make deductions, and justify results. The Core Skills are meant to be used strategically and adaptively to solve problems. Students' knowledge and skills come to life and take their value when melded with the ways they approach mathematics—as described by the Core Practices.

The specific verbs used to describe concepts and skills in these standards are not meant to limit or indicate levels of any taxonomy. Although using verbs to indicate levels of depth has been a common practice in this country's standards writing, high performing nations do not use verbs in this way. They describe depth and practices first in separate sections of their syllabi. We have adopted the high performing countries' practice of focusing on a clear statement of what mathematics should be learned when writing standards for knowledge and skills.

Instruction, curriculum and assessment designed to achieve these standards should range over all strands of proficiency in *Adding It Up*, all depths of knowledge in Norman L. Webb's Depth of Knowledge taxonomy, all levels of Bloom's Taxonomy, and all levels of cognitive demand. In the Core Skills and Core Practices we have sometimes used terms like "explore" to indicate a lighter treatment with a goal of awareness and experience rather than proficiency. We have used Example Tasks to show the depth of knowledge and deployment of skills expected.

From *Adding it up: Helping children learn mathematics* (National Research Council, 2001, p. 116):

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, we have chosen mathematical proficiency to capture what we believe is necessary for anyone to learn mathematics successfully. Mathematical proficiency, as we see it, has five components, or strands:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

These standards are measurable; that is, they are observable and verifiable through the broad spectrum of student performances that may be assessed during classroom observation, school-based examinations and large-scale testing. The *College and Career Readiness Standards for Mathematics* can guide the development of assessment frameworks that distribute the assessment responsibilities across multiple levels of the educational system: state, district, school and teacher.

Students reaching these levels will be prepared for non-remedial college mathematics courses and will be prepared for training programs for career-level jobs; however, the *College and Career Readiness Standards for Mathematics* should not be construed as grade twelve exit standards. Students interested in STEM fields, and those who wish to go beyond for other reasons, will need to reach these standards before their senior year in order to have time to include additional mathematics. A number of pathways for advanced learning are possible and may be integrated throughout the high school experience and beyond.

The Common Core State Standards Initiative

The *College and Career Readiness Standards for Mathematics* will anchor the next phase of the Common Core State Standards Initiative: development of K-12 Mathematics Standards. Those K-12 Standards are in turn expected to guide the development of a next generation of assessments, developed collaboratively by multiple states. The K-12 Mathematics Standards will serve as a guide and tool for aligning instruction, curriculum, assessment, teacher supports, and systems of accountability. To ensure alignment, the Standard for Mathematical Practice, the Standards for Mathematical Content, and the Example Tasks should all be taken into account.

Overview of the Mathematical Practice Standard

Attend to precision.
Construct viable arguments.
Make sense of complex problems and persevere in solving them.
Look for structure.
Look for and express regularity in repeated reasoning.
Make strategic decisions about the use of technological tools.

Overview of the Mathematical Content Standards

Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations on numbers and extend to operations in algebra.

Quantity. A quantity is an attribute of an object or phenomenon that can be specified using a number and a unit, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

Expressions. Expressions use numbers, variables and operations to describe computations. The rules of arithmetic, the use of parentheses and the conventions about order of operations assure that the computation has a well-determined value.

Equations. An equation is a statement that two expressions are equal. Solutions to an equation are the values of the variables in it that make it true.

Functions. Functions model situations where one quantity determines another. Because nature and society are full of dependencies, functions are important tools in the construction of mathematical models.

Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

Shape. From only a few axioms, the deductive method of Euclid generates a rich body of theorems about geometric objects, their attributes and relationships.

Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Probability. Probability assesses the likelihood of an event in a situation that involves randomness. It quantifies the degree of certainty that an event will happen as a number from 0 through 1.

Statistics. Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability in the data.

How Evidence Informed Decisions in Drafting the Standards

The Common Core State Standards Initiative builds on a generation of standards efforts led by states and national organizations. On behalf of the states, we have taken a step toward the next generation of standards that are aligned to college- and career-ready expectations and are internationally benchmarked. These standards are grounded in evidence from many sources that shows that the next generation of standards in mathematics must be focused on deeper, more thorough understanding of more fundamental mathematical ideas and higher mastery of these fewer, more useful skills.

The evidence that supports this new direction comes from a variety of sources. International comparisons show that high performing countries focus on fewer topics and that the U.S. curriculum is "a mile wide and an inch deep." Surveys of college faculty show the need to shift away from high school courses that merely survey advanced topics, toward courses that concentrate on developing an understanding and mastery of ideas and skills that are at the core of advanced mathematics. Reviews of data on student performance show the large majority of U.S. students are not mastering the mile wide list of topics that teachers cover.

The evidence tells us that in high performing countries like Singapore, the gap between what is taught and what is learned is relatively smaller than in Malaysia or the U.S. states. Malaysia's standards are higher than Singapore's, but their performance is much lower. One could interpret the narrower gap in Singapore as evidence that they actually use their standards to manage instruction; that is, Singapore's standards were set within the reach of hard work for their system and their population. Singapore's Ministry of Education flags its webpage with the motto, "Teach Less, Learn More." We accepted the challenge of writing standards that could work that way for U.S. teachers and students: By providing focus and coherence, we could enable more learning to take place at all levels.

However, a set of standards cannot be simplistically "derived" from any body of evidence. It is more accurate to say that we used evidence to inform our decisions. A few examples will illustrate how this was done.

For example, systems of linear equations are covered by all states, yet students perform surprisingly poorly on this topic when assessed by ACT. We determined that systems of linear equations have high coherence value, mathematically; that this topic is included by all high performing nations; and that it has moderately high value to college faculty. Result: We included it in our standards.

A different and more complex pattern of evidence appeared with families of functions. Again we found that students performed poorly on problems related to many advanced functions (trigonometric, logarithmic, quadratic, exponential, and so on). Again we found that a number of states cover them, even though college faculty rated them lower in value. High performing countries include this material, but with different degrees of demand. We decided that we had to carve a careful line through these topics so that limited teaching resources could focus where most important. We decided that students should develop deep understanding and mastery of linear and exponential functions. They should also have familiarity with other families of functions, and apply their algebraic, modeling and problem solving skills to them—but not develop in-depth technical mastery and understanding. Thus we defined two distinct levels of attention and identified which families of functions got which level of attention.

Why were exponential functions selected for intensive focus in the Functions standard instead of, say, quadratic functions? What tipped the balance was the high coherence value of exponential functions in supporting modeling and their wide utility in work and life. Quadratic functions were also judged to be well supported by expectations defined under Expressions and Equations.

These examples indicate the kind of reasoning, informed by evidence, that it takes to design standards aligned to the demands of college and career readiness in a global economy. We considered inclusion in international standards, requirements of college and the workplace, surveys of college faculty and the business community, and other sources of evidence. As we navigated these sometimes conflicting signals, we always remained aware of the finiteness of instructional resources and the need for deep mathematical coherence in the standards.

At the end of this document, there is a listing of a number of sources that played a role in the deliberations described above and more generally throughout the process to inform our decisions. A hyperlinked version of the bibliography can be found online at www.corestandards.org.

College and Career Readiness Standards for Mathematics

Mathematical Practice

Proficient students expect mathematics to make sense. They take an active stance in solving mathematical problems. When faced with a non-routine problem, they have the courage to plunge in and try something, and they have the procedural and conceptual tools to carry through. They are experimenters and inventors, and can adapt known strategies to new problems. They think strategically.

Students who engage in these practices discover ideas and gain insights that spur them to pursue mathematics beyond the classroom walls. They learn that effort counts in mathematical achievement.[a] These are practices that expert mathematical thinkers encourage in apprentices. Encouraging these practices in our students should be as much a goal of the mathematics curriculum as is teaching specific content topics and procedures.[b] Taken together with the Standards for Mathematical Content, they support productive entry into college courses or career pathways.

Core Practices · Students can and do:

1 Attend to precision.

Mathematically proficient students organize their own ideas in a way that can be communicated precisely to others, and they analyze and evaluate others' mathematical thinking and strategies noting the assumptions made. They clarify definitions. They state the meaning of the symbols they choose, are careful about specifying units of measure and labeling axes, and express their answers with an appropriate degree of precision. Rather than saying, "let v be speed and let t be time," they would say "let v be the speed in meters per second and let t be the elapsed time in seconds from a given starting time." They recognize that when someone says the population of the United States in June 2008 was 304,059,724, the last few digits indicate unwarranted precision.

2 Construct viable arguments.

Mathematically proficient students understand and use stated assumptions, definitions and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They break things down into cases and can recognize and use counterexamples. They use logic to justify their conclusions, communicate them to others and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

3 Make sense of complex problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They consider analogous problems, try special cases and work on simpler forms. They evaluate their progress and change course if necessary. They try putting algebraic expressions into different forms or try changing the viewing window on their calculator to get the information they need. They look for correspondences between equations, verbal descriptions, tables, and graphs. They draw diagrams of relationships, graph data, search for regularity and trends, and construct mathematical models. They check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?"

4 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern. For example, in $x^2 + 5x + 6$ they can see the 5 as $2 + 3$ and the 6 as 2×3. They recognize the significance of an existing line in a geometric figure and can add an auxiliary line to make the solution of a problem clear. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects. For example, by seeing $5 - 3(x - y)^2$ as 5 minus a positive number times a square, they see that it cannot be more than 5 for any real numbers x and y.[b]

5 Look for and express regularity in repeated reasoning.

Mathematically proficient students pay attention to repeated calculations as they carry them out, and look both for general algorithms and for shortcuts. For example, by paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, they might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel in the expansions of $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ leads to the general formula for the sum of a geometric series. As they work through the solution to a problem, proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.[b]

6 Make strategic decisions about the use of technological tools.

Mathematically proficient students consider the available tools when solving a mathematical problem, whether pencil and paper, ruler, protractor, graphing calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software. They are familiar enough with all of these tools to make sound decisions about when each might be helpful. They use mathematical understanding and estimation strategically, attending to levels of precision, to ensure appropriate levels of approximation and to detect possible errors. They are able to use these tools to explore and deepen their understanding of concepts.

(a) For the importance of students' beliefs about effort, see the National Mathematics Advisory Panel's Report of the Task Group on Learning Processes, p. 4-10 (2008). (b) Cuoco, A., Goldenberg, E. P., and Mark, J., *Journal of Mathematical Behavior*, 15 (4), 375-402, 1996; *Focus in High School Mathematics*. Reston, VA: NCTM, in press; Harel, G., What is Mathematics? A Pedagogical Answer to a Philosophical Question. In R. B. Gold & R. Simons (Eds.), *Current Issues in the Philosophy of Mathematics From the Perspective of Mathematicians*, Mathematical Association of America, 2008.

Number

Core Concepts · Students understand that:

A The real numbers include the rational numbers and are in one-to-one correspondence with the points on the number line.

B Quantities can be compared using division, yielding rates and ratios.

C A fraction can represent the result of dividing the numerator by the denominator; equivalent fractions have the same value.

D Place value and the rules of arithmetic form the foundation for efficient algorithms.

A Coherent Understanding of Number. Procedural fluency in operations with real numbers and strategic competence in approximation are grounded in an understanding of place value. The rules of arithmetic govern operations on numbers and extend to operations in algebra:

- Numbers can be added in any order with any grouping and multiplied in any order with any grouping.
- Adding 0 and multiplying by 1 both leave a number unchanged.
- All numbers have additive inverses, and all numbers except zero have multiplicative inverses.
- Multiplication distributes over addition.

Subtraction and division are defined in terms of addition and multiplication, so are also governed by these rules.

The place value system bundles units into 10s, then 10s into 100s, and so on, providing an efficient way to name large numbers. Subdividing in a similar way extends this to the decimal system, which provides an address system for locating all real numbers on the number line with arbitrarily high accuracy. Place value is the basis for efficient algorithms, reducing much computation to single-digit arithmetic. Mental computation strategies also make opportunistic use of the rules of arithmetic, as when the product $5\times177\times2$ is computed at a glance to obtain 1770, rather than methodically working from left to right.

An estimate may be more appropriate than an exact value, for example, when you want to know the number of calories in a meal. Often a result is reported using fewer digits than were calculated. A mature number sense includes having rules of thumb about how much accuracy is appropriate and understanding that accuracy to more than a few decimal places often takes substantial effort. Estimation and approximation are also useful in checking calculations.

Rational numbers represented as fractions can be located on the number line by seeing them as numbers expressed in different units; for example, 3/5 is 3 units, where each unit is 1/5. However, rational numbers do not fill out the number line. There are also irrational numbers, such as π or $\sqrt{2}$. Each point on the number line then corresponds to a real number that is either rational or irrational.

Connections to Expressions, Functions and Coordinates. The rules of arithmetic govern the manipulations of expressions and functions. Two perpendicular number lines define the coordinate plane.

Core Skills · Students can and do:

1 Compare numbers and make sense of their magnitude.

Include positive and negative numbers expressed as fractions, decimals, powers, and roots. Limit to square and cube roots. Include very large and very small numbers and the use of scientific notation.

2 Know when and how to use standard algorithms, and perform them flexibly, accurately and efficiently.*

3 Use mental strategies and technology to formulate, represent and solve problems.**

4 Solve multi-step problems involving fractions and percentages.

Include situations such as simple interest, tax, markups/markdowns, gratuities and commissions, fees, percent increase or decrease, percent error, expressing rent as a percentage of take-home pay, and so on.

5 Use estimation and approximation to solve problems.

Include evaluating answers for their reasonableness, detecting errors, and giving answers to an appropriate level of precision.

* This aligns with the concept of procedural fluency as in the National Research Council report *Adding it up: Helping children learn mathematics*. Specifically, "*Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121).

** This aligns with the concept of strategic competence as described in *Adding it up*. "*Strategic competence* refers to the ability to formulate mathematical problems, represent them, and solve them" (p. 124).

Quantity

Core Concepts · Students understand that:

A The value of a quantity is not specified unless the units are named or understood from the context.

B Quantities can be added and subtracted only when they are of the same type (length, area, speed, etc.).

C Quantities can be multiplied or divided to create new types of quantities, called derived quantities.

A Coherent Understanding of Quantity. A quantity is an attribute of an object or phenomenon that can be specified using a number and a unit, such as 2.7 centimeters, 42 questions or 28 miles per gallon.

The length of a football field and the speed of light are both quantities. If we choose units of miles per second, then the speed of light has a value of approximately 186,000 miles per second. But the speed of light need not be expressed in miles per second; it may be approximated by 3×10^8 meters per second or in any other unit of speed. Bare numerical values such as 186,000 do not describe quantities unless they are paired with units.

Speed (distance divided by time), rectangular area (length multiplied by length), density (mass divided by volume), and population density (number of people divided by land area) are examples of derived quantities, obtained by multiplying or dividing quantities.

It can make sense to add two quantities, such as when a child 51 inches tall grows 3 inches to become 54 inches tall. To be added or subtracted, quantities must be of the same type (length, area, speed, etc.); to add or subtract their values, the quantities must be expressed in the same units. Converting quantities to have the same units is like converting fractions to have a common denominator before adding or subtracting. But, even when quantities have the same units it does not always make sense to add them. For example, if a wooded park with 300 trees per acre is next to a field with 30 trees per acre, they do not have 330 trees per acre.

Doing algebra with units in a calculation reveals the units of the answer, and can help reveal a mistake if, for example, the answer comes out to be a distance when it should be a speed.

Connections to Number, Expressions, Equations, Functions, Modeling and Statistics. Operations described under Number and Expressions govern the operations one performs on quantities, including the units involved. Quantity is an integral part of any application of mathematics, and has connections to solving problems using data, equations, functions and modeling.

Core Skills · Students can and do:

1 Know when and how to convert units in computations.

Include the addition and subtraction of quantities of the same type expressed in different units; averaging data given in mixed units; converting units for derived quantities such as density and speed.

2 Use and interpret quantities and units correctly in algebraic formulas.

Include specifying units when defining variables and attending to units when writing expressions and equations.

3 Use and interpret quantities and units correctly in graphs and data displays.

Include function graphs, data tables, scatterplots and other visual displays of dimensioned data.

4 Use units as a way to understand problems and to guide the solution of multi-step problems.

Include examples such as acceleration; currency conversions; people-hours; social science measures, such as deaths per 100,000; and general rates, such as points per game.

Expressions

Core Concepts · Students understand that:

A Expressions are constructions built up from numbers, variables, and operations, which have a numerical value when each variable is replaced with a number.

B Complex expressions are made up of simpler expressions.

C The rules of arithmetic can be applied to transform an expression without changing its value.

D Rewriting expressions in equivalent forms serves a purpose in solving problems.

A Coherent Understanding of Expressions. Expressions use numbers, variables and operations to describe computations. The rules of arithmetic, the use of parentheses and the conventions about order of operations assure that the computation has a well-determined value.

Reading an expression with comprehension involves analysis of its underlying structure, which may suggest a different but equivalent way of writing it that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. But rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying by a constant factor.

Algebraic manipulations are based on the conventions of algebraic notation and the rules of arithmetic. Heuristic mnemonic devices are not a substitute for procedural fluency. For example, factoring, expanding, collecting like terms, the rules for interpreting minus signs next to parenthetical sums, and adding fractions with a common denominator are all instances of the distributive law; the definitions for negative and rational exponents are based on the extension of the exponent laws for positive integers. The laws of exponents connect multiplication of numbers to addition of exponents and thus express the deep relationship between addition and multiplication captured by the parallel nature of the rules of arithmetic for these operations.

Complex expressions are made up of simpler expressions using arithmetic operations and substitution. When simple expressions within more complex expressions are treated as single quantities, or chunks, the underlying structure of the larger expression may be more evident.

Connections to Equations and Functions. Setting expressions equal to each other leads to equations. Expressions can define functions of the variables that appear in them, with equivalent expressions defining the same function.

Core Skills · Students can and do:

1 See structure in expressions.

For example, recognize: that the expressions $x^4 - y^4$ and $(x + y)^2 - (x - y)^2$ are differences of squares; that there are different ways to rewrite the latter expression, e.g., by expanding and collecting like terms or by factoring as a difference of squares; that p is a common factor in $p + 0.025p$; that an expression in the form $(x - 3)^2 + 14$ reveals its minimum value.

2 Manipulate simple expressions.

Show procedural fluency in the following cases: factoring out common terms; factoring expressions with quadratic structure; writing in standard form sums, differences, and products of polynomials. Include completing the square and rewriting in standard form sums, differences, products, and quotients of simple rational expressions; rewriting expressions with negative exponents and those involving square or cube roots of a single term involving exponents.

3 Define variables and write an expression to represent a quantity in a problem.

Include contextual problems.

4 Interpret an expression that represents a quantity in terms of the context.

Include interpreting parts of an expression, such as terms, factors and coefficients.

Equations

Core Concepts · Students understand that:

A An equation is a statement that two expressions are equal.

B The solutions of an equation are the values of the variables that make the resulting numerical statement true.

C The steps in solving an equation are guided by understanding and justified by logical reasoning.

D Equations not solvable in one number system may have solutions in a larger number system.

A Coherent Understanding of Equations. An equation is a statement that two expressions are equal. Solutions to an equation are the values of the variables in it that make it true. If the equation is true for all values of the variables, then we call it an identity; identities are often discovered by manipulating one expression into another.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs, which can be graphed in the plane. Equations can be combined into systems to be solved simultaneously.

An equation can be solved by successively transforming it into one or more simpler equations. The process is governed by deductions based on the properties of equality. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, stimulating the formation of expanded number systems (integers, rational numbers, real numbers and complex numbers).

A formula is a type of equation. The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = \left(\frac{b_1+b_2}{2}\right)h$, can be solved for h using the same deductive process.

Inequalities can be solved in much the same way as equations. Many, but not all, of the properties of equality extend to the solution of inequalities.

Connections to Functions, Coordinates, and Modeling. Equations in two variables may define functions. Asking when two functions have the same value leads to an equation; graphing the two functions allows for the approximate solution of the equation. Equations of lines involve coordinates, and converting verbal descriptions to equations is an essential skill in modeling.

Core Skills · Students can and do:

1 Understand a problem and formulate an equation to solve it.

Extend to inequalities and systems.

2 Solve equations in one variable using manipulations guided by the rules of arithmetic and the properties of equality.

Solve linear equations with procedural fluency. For quadratic equations, include solution by inspection, by factoring, or by using the quadratic formula. Understand that the quadratic formula comes from completing the square. Include simple absolute value equations solvable by direct inspection and by understanding the interpretation of absolute value as distance.

3 Rearrange formulas to isolate a quantity of interest.

Exclude cases that require extraction of roots or inverse functions.

4 Solve systems of equations.

Focus on pairs of simultaneous linear equations in two variables. Include algebraic techniques, graphical techniques and solving by inspection.

5 Solve linear inequalities in one variable and graph the solution set on a number line.

Emphasize solving the associated equality and determining on which side of the solution of the associated equation the solutions to the inequality lie.

6 Graph the solution set of a linear inequality in two variables on the coordinate plane.

Emphasize graphing the associated equation, using a dashed or solid line as appropriate and shading to indicate the half-plane on which the solutions to the inequality lie.

Functions

Core Concepts · Students understand that:

A A function is a rule, often defined by an expression, that assigns a unique output for every input.

B The graph of a function f is a set of ordered pairs $(x, f(x))$ in the coordinate plane.

C Functions model situations where one quantity determines another.

D Common functions occur in families where each member describes a similar type of dependence.

A Coherent Understanding of Functions. Functions model situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because nature and society are full of dependencies between quantities, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a plane to fly 1000 miles is a function of the plane's average ground speed in miles per hour, v; the rule $T(v) = 1000/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of possible inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. The graph of a function is a useful way of visualizing the relationship the function models, and manipulating the expression for a function can throw light on the function's properties.

Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with an initial value of zero describe proportional relationships.

Connections to Expressions, Equations, Modeling and Coordinates. Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. The graph of a function f is the same as the solution set of the equation $y = f(x)$. Questions about when two functions have the same value lead to equations, whose solutions can be visualized from the intersection of the graphs. Since functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be modeled effectively using a spreadsheet or other technology.

Core Skills · Students can and do:

1 Recognize proportional relationships and solve problems involving rates and ratios.

Include being able to express proportional relationships as functions.

2 Describe the qualitative behavior of common types of functions using graphs and tables.

Identify: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Use technology to explore the effects of parameter changes on the graphs of linear, power, quadratic, polynomial, simple rational, exponential, logarithmic, sine and cosine, absolute value and step functions.

3 Analyze functions using symbolic manipulation.

Include slope-intercept and point-slope form of linear functions; vertex form of quadratic functions to identify symmetry and find maximums and minimums; factored form to find zeros. Use manipulations as described under Expressions.

4 Use the families of linear and exponential functions to solve problems.

For linear functions $f(x) = mx + b$, understand b as the intercept or initial value and m as the slope or rate of change. For exponential functions $f(x) = a \cdot b^x$, understand a as the intercept or initial value and b as the growth factor.

5 Find and interpret rates of change.

Compute the rate of change of linear functions and make qualitative observations about how the rate of change varies for nonlinear functions.

Modeling

Core Concepts · Students understand that:

A Mathematical models involve choices and assumptions that abstract key features from situations to help us solve problems.

B Even very simple models can be useful.

A Coherent Understanding of Modeling. Modeling uses mathematics to help us make sense of the real world—to understand quantitative relationships, make predictions, and propose solutions.

A model can be very simple, such as a geometric shape to describe a physical object like a coin. Even so simple a model involves making choices. It is up to us whether to model the solid nature of the coin with a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. For some purposes, we might even choose to adjust the right circular cylinder to model more closely the way the coin deviates from the cylinder.

In any given situation, the model we devise depends on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models we can create and analyze is constrained as well by the limitations of our mathematical and technical skills. For example, modeling a physical object, a delivery route, a production schedule, or a comparison of loan amortizations each requires different sets of tools. Networks, spreadsheets and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations. One of the insights provided by mathematical modeling is that essentially the same mathematical structure might model seemingly different situations.

The basic modeling cycle is one of (1) identifying the key features of a situation, (2) creating geometric, algebraic or statistical objects that describe key features of the situation, (3) analyzing and performing operations on these objects to draw conclusions and (4) interpreting the results of the mathematics in terms of the original situation. Choices and assumptions are present throughout this cycle.

Connections to Quantity, Equations, Functions, Shape, Coordinates and Statistics. Modeling makes use of shape, data, graphs, equations and functions to represent real-world quantities and situations.

Core Skills · Students can and do:

1 Model numerical situations.

Include readily applying the four basic operations in combination to solve multi-step quantitative problems with dimensioned quantities; making estimates to introduce numbers into a situation and get problems started; recognizing proportional or near-proportional relationships and analyzing them using characteristic rates and ratios.

2 Model physical objects with geometric shapes.

Include common objects that can reasonably be idealized as two- and three-dimensional geometric shapes. Identify the ways in which the actual shape varies from the idealized geometric model.

3 Model situations with equations and inequalities.

Include situations well described by a linear inequality in two variables or a system of linear inequalities defining a region in the plane.

4 Model situations with common functions.

Include situations well described by linear, quadratic or exponential functions; and situations that can be well described by inverse variation ($f(x) = k/x$). Include identifying a family of functions that models features of a problem, and identifying a particular function of that family and adjusting it to fit by changing parameters. Understand the recursive nature of situations modeled by linear and exponential functions.

5 Model situations using probability and statistics.

Include using simulations to model probabilistic situations; describing the shape of a distribution of values and summarizing a distribution with measures of center and variability; modeling a bivariate relationship using a trend line or a regression line.

6 Interpret the results of applying a model and compare models for a particular situation.

Include realizing that models seldom fit exactly and so there can be error; identifying simple sources of error and being careful not to over-interpret models. Include recognizing that there can be many models that relate to a situation, that they can capture different aspects of the situation, that they can be simpler or more complex, and that they can have a better or worse fit to the situation and the questions being asked.

Shape

Core Concepts · Students understand that:

A Shapes and their parts, attributes, and measurements can be analyzed deductively.*

B Congruence, similarity, and symmetry can be analyzed using transformations.

C Mathematical shapes model the physical world, resulting in practical applications of geometry.

D Right triangles and the Pythagorean theorem are central to geometry and its applications, including trigonometry.

A Coherent Understanding of Shape. From only a few axioms, the deductive method of Euclid generates a rich body of theorems about geometric objects, their attributes and relationships. Once understood, those attributes and relationships can be applied in diverse practical situations—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Understanding the attributes of geometric objects often relies on measurement: a circle is a set of points in a plane at a fixed distance from a point; a cube is bounded by six squares of equal area; when two parallel lines are crossed by a transversal, pairs of corresponding angles are congruent.

The concepts of congruence, similarity and symmetry can be united under the concept of geometric transformation. Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent. Applying a scale transformation to a geometric figure yields a similar figure. The transformation preserves angle measure, and lengths are related by a constant of proportionality. If the constant of proportionality is one, distances are also preserved (so the transformation is a rigid transformation) and the figures are congruent.

The definitions of sine, cosine and tangent for acute angles are founded on right triangle similarity, and, with the Pythagorean theorem, are fundamental in many practical and theoretical situations.

Connections to Coordinates, Functions and Modeling. The Pythagorean theorem is a key link between geometry, measurement and distance in the coordinate plane. Parameter changes in families of functions can be interpreted as transformations applied to their graphs and those functions, as well as geometric objects in their own right, can be used to model contextual situations.

*In this document, deductive analysis aligns with the notion of adaptive reasoning as defined in *Adding it Up*, and includes empirical exploration, informal justification, and formal proof.

Core Skills · Students can and do:

1 Use multiple geometric properties to solve problems involving geometric figures.

Properties include: measures of interior angles of a triangle sum to 180°; vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; measures of supplementary angles sum to 180°; two lines parallel to a third are parallel to each other; points on a perpendicular bisector of a segment are exactly those equidistant from the segment's endpoints; and a line tangent to a circle is perpendicular to the radius meeting it.

2 Prove theorems, test conjectures and identify logical errors.

Include theorems establishing the properties in Core Skill 1 and other theorems about angles, parallel and perpendicular lines, similarity and congruence of triangles.

3 Construct and interpret representations of geometric objects.

Include classical construction techniques and construction techniques supported by modern technologies. Include moving between two-dimensional representations and the three-dimensional objects they represent, such as in schematics, assembly instructions, perspective drawings and multiple views.

4 Solve problems involving measurements.

Include measurement (length, angle measure, area, surface area, and volume) of a variety of figures and shapes in two- and three-dimensions. Compute measurements using formulas and by decomposing complex shapes into simpler ones.

5 Solve problems involving similar triangles and scale drawings.

Include computing actual lengths, areas and volumes from a scale drawing and reproducing a scale drawing at a different scale.

6 Apply properties of right triangles and right triangle trigonometry to solve problems.

Include using the Pythagorean theorem and properties of special right triangles, and applying sine, cosine and tangent to determine lengths and angle measures of right triangles. Use right triangles and their properties to solve real-world problems. Limit angle measures to degrees.

Coordinates

Core Concepts · Students understand that:

A Locations in the plane or in space can be specified by pairs or triples of numbers called coordinates.

B Coordinates link algebra with geometry and allow methods in one domain to solve problems in the other.

C The set of solutions to an equation in two variables forms a curve in the coordinate plane—such as a line, parabola, circle—and the solutions to systems of equations correspond to intersections of these curves.

A Coherent Understanding of Coordinates. Applying a coordinate system to Euclidean space connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling and proof.

Coordinate geometry is a rich field for exploration. How does a geometric transformation such as a translation or reflection affect the coordinates of points? How is the geometric definition of a circle reflected in its equation?

Adding a third perpendicular axis associates three numbers with locations in three dimensions and extends the use of algebraic techniques to problems involving the three-dimensional world we live in.

Connections to Shape, Quantity, Equations and Functions. Coordinates can be used to reason about shapes. In applications, coordinate values often have units (such as meters and bushels). A one-variable equation of the form $f(x) = g(x)$ may be solved in the coordinate plane by finding intersections of the curves $y = f(x)$ and $y = g(x)$.

Core Skills · Students can and do:

1 Translate fluently between lines in the coordinate plane and their equations.

Include predicting visual features of lines by inspection of their equations, determining the equation of the line through two given points, and determining the equation of the line with a given slope passing through a given point.

2 Identify the correspondence between parameters in common families of equations and the location and appearance of their graphs.

Include common families of equations—the graphs of $Ax + By = C$, $y = mx + b$ and $x = a$ are straight lines; the graphs of $y = a(x - h)^2 + k$ and $y = Ax^2 + Bx + C$ are parabolas; and the graph of $(x - h)^2 + (y - k)^2 = r^2$ is a circle.

3 Use coordinates to solve geometric problems.

Include proving simple theorems algebraically, using coordinates to compute perimeters and areas for triangles and rectangles, finding midpoints of line segments, finding distances between pairs of points and determining when two lines are parallel or perpendicular.

Probability

Core Concepts · Students understand that:

A Probability models outcomes for situations in which there is inherent randomness, quantifying the degree of uncertainty in terms of relative frequency of occurrence.

B The law of large numbers provides the basis for estimating certain probabilities by use of empirical relative frequencies.

C The laws of probability govern the calculation of probabilities of combined events.

D Interpreting probabilities contextually is essential to rational decision-making in situations involving randomness.

A Coherent Understanding of Probability. Probability assesses the likelihood of an event in a situation that involves randomness. It quantifies the degree of certainty that an event will happen as a number from 0 through 1. This number is generally interpreted as the relative frequency of occurrence of the event over the long run.

The structure of a probability model begins by listing or describing the possible outcomes for a random situation (the sample space) and assigning probabilities based on an assumption about long-run relative frequency. In situations such as flipping a coin, rolling a number cube, or drawing a card, it is reasonable to assume various outcomes are equally likely.

Compound events constructed from these simple ones can be represented by tree diagrams and by frequency or relative frequency tables. The probabilities of compound events can be computed using these representations and by applying the additive and multiplicative laws of probability. Interpreting these probabilities relies on an understanding of independence and conditional probability, approachable through the analysis of two-way tables.

Converting a verbally-stated problem into the symbols and relations of probability requires careful attention to words such as *and*, *or*, *if*, and *all*, and to grammatical constructions that reflect logical connections. This is especially true when applying probability models to real-world problems, where simplifying assumptions are also usually necessary in order to gain at least an approximate solution.

Connections to Statistics and Expressions. Probability is the foundation for drawing valid conclusions from sampling or experimental data. Counting has an advanced connection with Expressions through Pascal's triangle and binomial expansions.

Core Skills · Students can and do:

1 Compute theoretical probabilities by systematically counting points in the sample space.

Make use of symmetry and equally likely outcomes. Include permutation and combination problems as long as small numbers are involved or technology is used, so that formulas are not required.

2 Interpret probabilities of compound events using concepts of independence and conditional probability.

Include reading conditional probabilities from two-way tables.

3 Compute probabilities of compound events.

Make use of the additive and multiplicative laws of probability, tree diagrams and frequency or relative frequency tables in real contexts. Do not emphasize fluency with the related formulas

4 Estimate probabilities empirically.

Include using data from simulations carried out with technology to estimate probabilities.

5 Identify and explain common misconceptions regarding probability.

Include misconceptions about long-run versus short-run behavior of relative frequencies (the law of large numbers). Include attention to the use and misuse of probability in the media, especially in terms of interpreting charts and tables and in the contextual meaning of terms connected to probability, such as 'odds' or 'risk.'

6 Adapt probability models to solve real-world problems.

Include the use of conditional probability to assess subsets of data (e.g., what does the data say about males and females separately). Include the use of independence as a simplifying assumption (e.g., find the probability that two students both contract the disease this year).

Statistics

Core Concepts · Students understand that:

A Statistical methods take variability into account to support making informed decisions based on quantitative studies designed to answer specific questions.

B Visual displays and summary statistics condense the information in data sets into usable knowledge.

C Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic.

D The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

A Coherent Understanding of Statistics. Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability in the data. Statistics provides tools for describing variability in data and for making informed decisions that take variability into account.

Data are gathered, displayed, summarized, examined and interpreted to discover patterns. Data can be summarized by a statistic measuring center, such as mean or median, and a statistic measuring spread, such as interquartile range or standard deviation. Different distributions can be compared numerically using these statistics or visually using plots. Which statistics to compare, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance and this can be evaluated only under the condition of randomness.

In critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were collected, and the analyses employed as well as the data summaries and the conclusions drawn.

Connections to Probability, Functions and Modeling. Valid conclusions about a population depend on designed simulations or other statistical studies using random sampling or assignment and rely on probability for their interpretation. Functional models may be used to approximate data. If the data are approximately linear, the relationship may be modeled with a trend line and the strength and direction of such a relationship may be expressed through a correlation coefficient. Technology facilitates the study of statistics by making it possible to simulate many possible outcomes in a short amount of time, and by generating plots, function models, trend lines and correlation coefficients.

Core Skills · Students can and do:

1 Formulate questions that can be addressed with data. Identify the relevant data, collect and organize it to respond to the question.

Include determining whether a question can best be addressed through a sample survey, randomized experiment or observational study. Include unbiased selection for a sample and randomization of assignment to treatment for an experiment.

2 Use appropriate displays and summary statistics for data.

Include univariate, bivariate, categorical and quantitative data. Include the thoughtful selection of displays and measures of center and spread to summarize data.

3 Interpret data displays and summaries critically; draw conclusions and develop recommendations.

Include paying attention to the context of the data, interpolating or extrapolating judiciously, and examining the effects of extreme values of the data on summary statistics of center and spread. Include data sets that follow a normal distribution. Include observing and interpreting linear trends in bivariate quantitative data.

4 Draw statistical conclusions involving population means or proportions using sample data.

Conclusions should be based on simulations or other informal techniques, rather than formulas.

5 Evaluate reports based on data.

Include looking for bias or flaws in the way the data were gathered or presented, as well as unwarranted conclusions, such as claims that confuse correlation with causation.

Sample of Works Consulted

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Quantity

Core Concepts

A The value of a quantity is not specified unless the units are named or understood from the context.

B Quantities can be added and subtracted only when they are of the same type (length, area, speed, etc.).

C Quantities can be multiplied or divided to create new types of quantities, called derived quantities.

Core Skills

1. Know when and how to convert units in computations.
2. Use and interpret quantities and units correctly in algebraic formulas.
3. Use and interpret quantities and units correctly in graphs and data displays.
4. Use units as a way to understand problems and to guide the solution of multistep problems.

Example Tasks

1. **Core Concept A.**

   The following question was posted on an Internet gardening forum:

   > I am trying to figure out how many yards of soil I need for my gardens but have no idea how much a yard of soil actually is. The nursery says they deliver 6 yards in a dump truck. I realize that a yard is 3 feet. But what is a yard of soil—is that 3 feet long and 3 feet high? I'm clueless! LOL

   Write a helpful response to the person who posted this question.

2. **Core Concept B, Core Skill 2.**

   A textbook printed the following formula for the surface area of a cylinder:

   $$SA = 2\pi r + 2\pi rh\,.$$

   Kim had never studied geometry, but she knew there must be a typographical error in this formula. How could she tell?

3. **Core Concept B, Core Skill 4.**

   As a flooring contractor, Lupe sets floor tile for a living. She submits a bid for each new job. When preparing a bid, she measures the area of the floor to be tiled and then figures out how much material she will need. She charges the following prices for materials and labor:

   - Subflooring: $1.27 per square foot
   - Tile: $6.59 per square foot
   - Adhesive: $31.95 per job
   - Grout: $55.95 per job
   - Labor: $125 base price plus $0.79 per square foot

   For a job tiling an area of 550 square feet, what is the amount of the bid, based on the materials listed above?

4. **Core Concept C.**

   If there are 8×10^{12} hydrogen molecules in a volume of 4×10^4 cubic centimeters, what is the average number of hydrogen molecules per cubic centimeter?

5. **Core Concept C.**

   The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer?

6. **Core Skill 1.**

   Signs in Canada give the speed limit in kilometers per hour. If the speed limit is 100 km/hr, and you're driving at a speed of 65 mph, are you over or under the speed limit? By how much?

7. **Core Skill 1.**

   Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?

8. **Core Skill 1; Core Skill 4; Number Core Skill 2; Number Core Skill 3; Number Core Skill 5.**

   A doctor orders Ceclor elixir for a child who weighs 9.3 kg. The child must receive 25 mg of the drug for each kilogram of body weight. The hospital pharmacy stocks Ceclor elixir in a concentration of 250 mg per 5 ml.

How many milliliters of the stock elixir should the child receive?

(a) Estimate the answer mentally. (Suggestion: approximate the child's weight as 10 kg.)
(b) Compute the answer to the nearest tenth of a milliliter.

9. **Core Skill 2, Core Skill 4.**

The distance traveled by a freely falling object dropped from rest is given by the formula $s = \frac{1}{2} g t^2$. Here s is the distance fallen, g is a constant representing the earth's surface gravity, and t is the duration of time over which the object falls. If s has units of meters and t has units of seconds, what must be the units of g? If we interpret g as a rate of change, what sort of quantity is changing with time? What does this mean about the speed of a freely falling object?

10. **Core Skill 2; Equations Core Skill 3.**

The *ideal gas law* is a mathematical relationship that describes the pressure, volume, and temperature of an ideal gas in thermal equilibrium:

$$pV = nRT.$$

In the ideal gas law, p stands for pressure, V stands for volume, n is the number of moles of gas, R is a constant equal to $0.0821 \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}$, and T stands for temperature measured in the Kelvin temperature scale.

(a) Find the temperature of 0.520 mol of argon gas occupying a volume of 4.25 L at a pressure of 0.987 atm.
(b) If the volume stays the same as in (a), but the pressure drops by half, then what must have happened to the temperature?

11. **Core Skill 4.**

According to *Runner's World* magazine,

> On average the human body is more than 50 percent water. Runners and other endurance athletes average around 60 percent. This equals about 120 soda cans' worth of water in a 160-pound runner!

Check out the *Runner's World* calculation. Are there really about 120 soda cans' worth of water in the body of a 160-pound runner? Here are some facts:

- A typical soda can holds 12 fluid ounces.
- 16 ounces (one pint) of water weighs one pound.

12. **Core Skill 4.**

A liquid weed-killer comes in four different kinds of bottles. The accompanying table gives information about the concentration, size, and price of the bottles. The "concentration" refers to the percent of "active ingredient" in the bottle. The rest of the liquid in the bottle is water. For example, bottle B contains $(0.18)(32) = 5.76$ fl. oz. of active ingredient and $32 - 5.76 = 26.24$ fl. oz. water.

Table 1 *Weed killer comes in four different kinds of bottles.*

	Concentration	Amount in Bottle	Price of Bottle
A	0.96%	64 fl. oz.	$12.99
B	18.0%	32 fl. oz.	$29.99
C	41.0%	32 fl. oz.	$39.99
D	0.96%	24 fl. oz.	$5.99

(a) Rank the four bottles in order of how good a buy each represents. State what criterion you are using.
(b) Suppose a job calls for a total of 12 fl. oz. of active ingredient. How much would you need to spend if you bought Type A bottles? Type B bottles? Type C bottles? Type D bottles?

13. **Core Skill 4.**

A table in a construction manual lists the "k-values" of different building materials. The k-value measures how easily heat flows through a material. The k-value of concrete is given as

$$0.002 \frac{\text{BTU}\cdot\text{in}}{\text{hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}}$$

A BTU is a unit of heat energy.

The construction manual gives the following example problem illustrating the k-value:

> How many BTUs of heat energy would be lost through a 100 ft^2 concrete wall 6 inches thick over a 12 hour period, if the temperature difference from one side of the wall to the other is 70°F?

Your friend Anders doesn't know much about construction—or about heat loss—yet he was able to get the answer, 28 BTUs, just by thinking about units. How did he get the answer?

14. **Core Skill 3.**

Each graph in the accompanying figure shows the relationship between distance traveled and time for a different train (Train A, Train B, and Train C).

Which train was traveling fastest during the interval of time shown? Justify your answer.

Distance Traveled Vs. Time

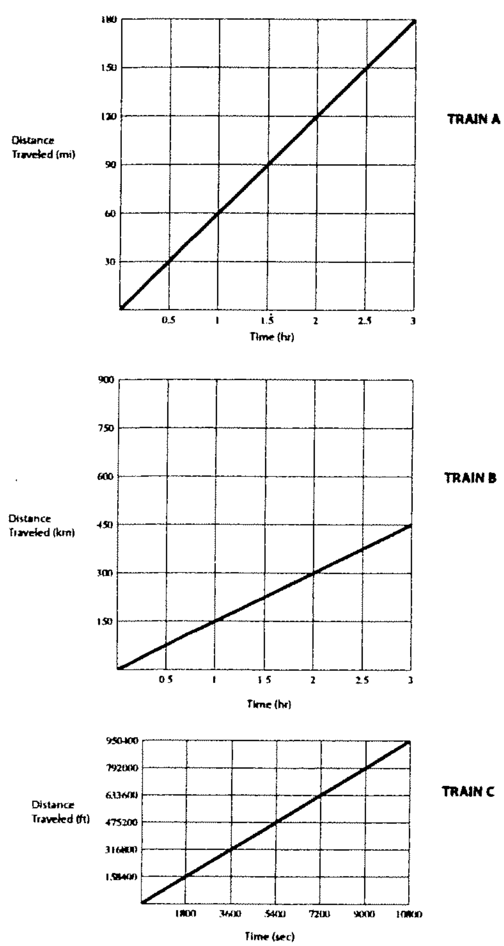

Figure 1

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Expressions

Core Concepts

A Expressions are constructions built up from numbers, variables, and operations, which have a numerical value when each variable is replaced with a number.

B Complex expressions are made up of simpler expressions.

C The rules of arithmetic can be applied to transform an expression without changing its value.

D Rewriting expressions in equivalent forms serves a purpose in solving problems.

Core Skills

1. See structure in expressions.
2. Manipulate simple expressions.
3. Define variables and write an expression to represent a quantity in a problem.
4. Interpret an expression that represents a quantity in terms of the context.

Example Tasks

1. **Core Concept A.**

   It is given that p = 4×10^5 and $q = 8 \times 10^6$. Expressing your answers in standard form, find

   (a) $\frac{p}{q}$.
   (b) $\sqrt[3]{q}$.

2. **Core Concept A.**

   Replace x by 16 in each of the following expressions and then simplify to an exact expression without using a calculator.

   (a) $3x - 48$
   (b) $3(x - 48)$
   (c) $\sqrt{x+9}$
   (d) $\sqrt{x} + \sqrt{9}$
   (e) $|x - 20| + (x - 20)$

3. **Core Concept A.**

   Simplify:

   (a) $3(x-2) + x[4 - 3(x-7)]$
   (b) $\frac{(a^3)^5}{(a^7)^2}$

   Partially check your answers by making appropriate calculations using $x = -5$, $x = 3$, $a = 2$, $a = -2$.

4. **Core Concept A.**

   It is given that $x = a + \sqrt{a^2 + b^2}$.

   (a) Calculate x when $a = 0.73$ and $b = 1.84$. Give your answer correct to 2 decimal places.
   (b) Express b in terms of x and a.

5. **Core Concept A, Core Skill 3.**

   Write an expression representing each of the following sequences of operations on a number x.

   - Multiply x by 5 then add 1.
   - Add 8 to x then multiply your answer by 5
   - Add 3 to x then divide your answer by 4
   - Multiply x by x then multiply your answer by 7
   - Multiply x by 3 then square your answer

6. **Core Concept A, Core Concept B, Core Skill 2, Core Skill 3, Core Skill 4.**

   As a flooring contractor, Lupe sets floor tile for a living. She submits a bid for each new job. When preparing a bid, she measures the area of the floor to be tiled and then figures out how much material she will need. She charges the following

prices for materials and labor:
Subflooring: \$1.27 per square foot

Tile: \$6.59 per square foot
Adhesive: \$31.95 per job
Grout: \$55.95 per job
Labor: \$125 base price plus \$0.79 per square foot.

Write an expression to determine the total cost of the materials and labor for a typical job in terms of the number of square feet. Explain what the numbers and symbols in the expression mean.

7. **Core Concept B.**

Describe the parts of each expression.

(a) $\left(\frac{l_1 + l_2}{2}\right) w$ (b) $(a-b)^2-(a+b)^2$

(c) $\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$.

COMMENTS ON SOLUTION:

For example, the expression in (a) is a product of two factors, the first of which is a quotient where the numerator is a sum.

8. **Core Concept C; Core Concept D; Core Skill 3; Core Skill 4.**

Ask the apprentice to mix a solution (#1) of 5 g Peters fertilizer and 50 g of distilled water.

- Determine the percent concentration-by-weight of this solution.

  The basic formula is *Weight of the solute divided by the combined weight of the solute and solvent equals percent concentration-by-weight.*

  Example: $5g \div (5g + 50g) = 5g \div 55g \approx 0.09$ or 9% concentration-by-weight.
- Calculate the density of this solution (#1).

  Divide the weight by the volume to determine the density in gm/ml.

Ask the apprentice to make a solution (#2) using 10 g of Peters and 50 g of distilled water.

- Determine the percent concentration-by-weight.
- Ask the apprentice: Why is the concentration-by-weight of solution #2 not double the concentration-by-weight of solution #1 since the solute is doubled?

$$C = \frac{x}{x+V} \quad \text{and} \quad \frac{2x}{2x+V} \neq 2\frac{x}{x+V}.$$

- Ask the apprentice to use the formula to explain.

9. **Core Concept C.**

Simplify each expression without evaluating each term. (Use the rules of arithmetic.)

(a) $12 \times 17 - 3 \times 24 - 6 \times 22$
(b) $(4 \times 17 + 3 \times 12) - 2 \times (12 + 17)$
(c) $(4a + 3b) - 2(b + a) + 1 - b$
(d) $(20 + 1) \times (20 - 1)$
(e) $(a + 1)(a - 1)$

10. **Core Concept C; Core Concept D; Core Skill 1; Core Skill 4.**

Which calculations show the correct number of small squares in Figure 1? Draw diagrams to explain the meaning of the other expressions.

(a) $4+6\times4$ (b) $4\times4+6$ (c) $4\times(4+6)$
(d) 4^2+6 (e) $4\times2+6$ (f) $4^2+4\times6$
(g) $4\times2+4\times6$

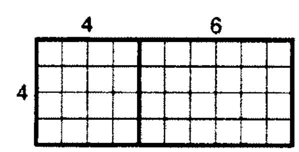

Figure 1

11. **Core Concept D.**

Evaluate

(a) $a + 2b + 3(a + b) + 2a$ when $a = -\frac{13}{3}, b = \frac{7}{5}$
(b) $7c - 2d + 3 - 5c - d - 2c$ when $c = \frac{9}{2}, d = -1$
(c) $3x+y+3-3(x+y)$ when $x = 1, y = -1$

COMMENTS ON SOLUTION:

Note that it can advantageous to expand and rearrange terms before evaluating.

12. **Core Concept D; Core Skill 1.**

Use Figure 2 to explain why the following expressions are equivalent:

$$(n+2)^2 - 4 \quad \text{and} \quad n^2 + 4n$$

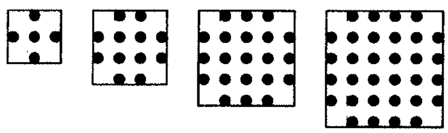

Figure 2

13. **Core Concept D; Core Skill 1.**

Write a possible identity for Line n, and show how it fits the pattern for each line in the list given.

(a)
- Line 1: $1 \times 3 = 2 + 1$
- Line 2: $2 \times 4 = 4 + 4$
- Line 3: $3 \times 5 = 6 + 9$
- Line 4 $4 \times 6 = 8 + 16$
- Line 5: $5 \times 7 = 10 + 25$

(b)
- Line 1: $1 - 1 = 2 \times 0$
- Line 2: $4 - 1 = 3 \times 1$
- Line 3: $9 - 1 = 4 \times 2$
- Line 4: $16 - 1 = 5 \times 3$
- Line 5: $25 - 1 = 6 \times 4$

14. **Core Concept D; Core Skill 1.**

The height of a thrown horseshoe depends upon the time that has elapsed since its release. Its height (measured in feet) as a function of time (measured in seconds) from the instant of release is

$$1\frac{3}{16} + 18t - 16t^2.$$

The expressions (a)–(d) below are equivalent. Which is most useful for finding the maximum height of the horseshoe's path, and why is it the most useful expression?

(a) $1\frac{3}{16} + 18t - 16t^2$

(b) $-16\left(t - \frac{19}{16}\right)\left(t + \frac{1}{16}\right)$

(c) $\frac{1}{16}(19 - 16t)(16t + 1)$

(d) $-16\left(t - \frac{9}{16}\right)^2 + \frac{100}{16}$.

15. **Core Skill 1.**

Which of the following is a factor of the polynomial below?

$$4x^3y - 8x^2y^2 + 10xy^3$$

(a) $4y^2$	(b) $2x^2$
(c) $2xy$	(d) x^2y^2

16. **Core Skill 1.**

Factorise completely: $6cx - 3cy - 2dx + dy$.

17. **Core Skill 1.**

A physics professor says: "Of course, it is easy to see that $L_0\sqrt{1 - \frac{v^2}{c^2}} = 0$ when $v = c$." Give a possible explanation in terms of the structure of this expression why the professor might say that.

18. **Core Skill 1.**

Give an explanation in terms of the structure of the expression

$$\frac{s}{\sqrt{n}}$$

why it halves in value when n is quadrupled.

19. **Core Skill 1.**

Which of (i)–(iv) would be the most productive first step in factoring expressions (a)–(f)?

(i) Finding a common factor
(ii) Expressing as a difference of two squares
(ii) Expressing as a perfect square
(iv) None of the above.

(a) $4x^2 + 5x$	(b) $36x^2 - 25y^2$
(c) $x^4 - y^4$	(d) $x^2 + 6xy + 9y^2$
(e) $x^2 + 5x + 4$	(f) $0.5x^2 - 2.5x - 7$

20. **Core Skill 1.**

When $(3u^2+au+5)(u^{10}+2u^4-bu^2)$ is written as a polynomial in u,

(a) what is the highest degree?

(b) for which powers of u are the terms nonzero?

COMMENTS ON SOLUTION:

Terms occur in degrees 2, 3, 4, 5, 6, 10, 11, 12 and the total (largest) degree is 12. This problem reinforces the structure-exploiting scanning procedure for finding products of sums with almost no arithmetic. Qualitative information of this sort is used to focus or avoid quantitative work.

21. **Core Skill 1.**

Find the y^3 term in

$$(3y^3 - y^2 + 5ay)(y^2 + 2y - a)$$

when it is written as a polynomial in y.

COMMENTS ON SOLUTION:

This term comes from products of a summand from the first factor and a summand from the second that give total y exponent 3. The y^3 is fixed and can be factored out ahead of time. Scanning and picking out the terms of interest gives

$$y^3\left[(3)(-a) + (-1)(2) + (5a)(1)\right]$$

- The expression just above has not been simplified, illustrating the idea that organizational and arithmetic tasks should be separated.
- Computing the whole polynomial and discarding unneeded terms requires far more work. The problem formulation therefore provides an incentive to understand the qualitative structure of products of sums.

22. **Core Skill 1.**

If $a \neq b$ and $\frac{1}{x} + \frac{1}{a} = \frac{1}{b}$, then $x =$

(a) $\frac{1}{b} - \frac{1}{a}$ (b) $b - a$

(c) $\frac{1}{ab}$ (d) $\frac{a-b}{ab}$

(e) $\frac{ab}{a-b}$

COMMENTS ON SOLUTION:

Although the problem can be solved by routine manipulation, it is quicker to solve for a student who sees structural reasons for eliminating some or all of the wrong choices.

23. **Core Skill 2.**

Factorise completely

(a) $15a^2 + 12a^3$,

(b) $1 - 16b^2$.

24. **Core Skill 2.**

Perform manipulations such as the following with procedural fluency:

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

25. **Core Skill 2.**

Simplify

$$\frac{a^{-2}b^2c}{a^2b^{-3}c^2}$$

26. **Core Skill 2.**

Simplify $\sqrt{x^9t^6}$ for $x > 0, t > 0$.

27. **Core Skill 3.**

A shopkeeper sells pens and pencils. Each pen costs \$5 and each pencil costs \$3. One day he sold x pens. On the same day he sold 9 more pens than pencils.

(a) Write down an expression, in terms of x, for his total income from the sale of these pens and pencils.

(b) This total income was less than \$300. Form an inequality in x and solve it.

(c) Hence write down the maximum number of pens that he sold.

28. **Core Skill 3.**

Suppose x adults and y children visit a theme park one day. The price of an adult ticket is a dollars and the price of a child's ticket is b dollars. Write an algebraic expression for:

(a) The total number of adults and children who visit the park that day.

(b) The cost in dollars of 2 adult tickets and 3 child tickets.

29. **Core Skill 3.**

Sam makes and sells mirrors. She makes a frame for each mirror with small square ceramic tiles. Find an expression for the number of tiles needed to border a size n frame.

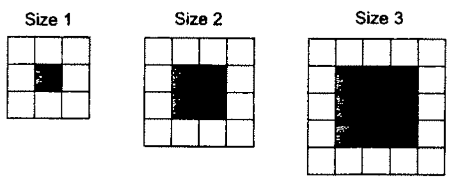

Figure 3

30. **Core Skill 3.**

A soccer tournament is being arranged. A number of teams have entered. Every team will play every other team twice: once at home and once away. Find an expression for the number of matches that will need to be played if N teams enter the tournament. Explain how you can be sure that your expression will work for all values of N.

COMMENTS ON SOLUTION:

There are N locations where games are played, one for each team. Furthermore, the number of games played at each location is $N - 1$, because each team plays a home game with every other team. Thus the total number of games is $N(N - 1)$.

31. **Core Skill 3; Core Skill 4.**

Consider the following statement about a positive integer n: The sum of any n consecutive integers is divisible by n.

For which values of n is the statement true? For which is it false? Prove that your answers are correct.

32. **Core Skill 3, Core Skill 4, Mathematical Practise 2.**

Prove that the sum of two odd integers is an even integer.

COMMENTS ON SOLUTION:

Let n and m be two odd integers. Since n is odd, it is one more than an even integer, so $n = 2k + 1$ for some integer k. By the same reasoning, $m = 2l + 1$ for some integer l. Therefore, the sum of n and m is

$$\begin{aligned} n + m &= (2k + 1) + (2l + 1) \\ &= 2k + 2l + 2 = 2(k + l + 1). \end{aligned}$$

So $n + m$ is 2 times the integer $k + l + 1$, and therefore it is even.

33. **Core Skill 4.**

Suppose x adults and y children visit a theme park one day. The price of an adult ticket is a dollars and the price of a child's ticket is b dollars. Interpret the following in everyday language:

(a) $a = 3b$

(b) $ax + by < 1000$

34. **Core Skill 4.**

In (a)–(d), give all values of a and n satisfying the condition, where a is real and n is a positive integer.

(a) $-a$ is positive

(b) $\sqrt{5 - a}$ is defined

(c) $\frac{1}{a}$ is an integer

(d) a^n is negative

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Equations

Core Concepts

A An equation is a statement that two expressions are equal.

B The solutions of an equation are the values of the variables that make the resulting numerical statement true.

C The steps in solving an equation are guided by understanding and justified by logical reasoning.

D Equations not solvable in one number system may have solutions in a larger number system.

Core Skills

1. Understand a problem and formulate an equation to solve it.
2. Solve equations in one variable using manipulations guided by the rules of arithmetic and the properties of equality.
3. Rearrange formulas to isolate a quantity of interest.
4. Solve systems of equations.
5. Solve linear inequalities in one variable and graph the solution set on a number line.
6. Graph the solution set of a linear inequality in two variables on the coordinate plane.

Example Tasks

1. **Core Concept A.**

   Are the following equations?

   (a) $y = x^2 + 3x + 2$
   (b) $x^2 + 3x + 2 = 0$
   (c) $x^2 + 3x + 2 = (x+1)(x+2)$
   (d) $\left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$
   (e) $x = -2$

2. **Core Concept B.**

   What are the solutions of the equation below?

   $$2n(3n - 12) = 0$$

   (a) 0 and 4 (b) 0 and 12
   (c) 2 and 4 (d) 2 and 12

3. **Core Concept B, Core Skill 6.**

   The shaded region inside the triangle ABC is defined by three inequalities. One of these is $x + y < 5\frac{1}{2}$.

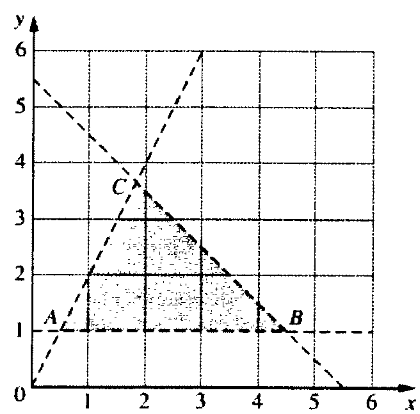

Figure 1

   (a) Write down the other inequalities.
   (b) How many points, with integer coordinates, lie in the shaded region?

4. **Core Concept B.**

   In (a)–(c), does the equation have a solution? Give a reason for your answer that does not depend on solving the equation.

(a) $\frac{t+2}{3+t} = 1$ (b) $\frac{3+t}{3-t} = 1$

(c) $\frac{t-2}{2-t} = 1$

5. **Core Concept C.**

A student performs the following steps in solving an equation:

$$\frac{x+3}{2x+6} = 1$$
$$x + 3 = 2x + 6$$
$$x = -3$$

Is the solution correct? If yes, explain why. If no, explain what was wrong with the student's reasoning.

6. **Core Concept C.**

If the equations

$$3x + 2y + 2z = 19$$
$$3x + y + z = 14$$

are true, which of the following is the value of $y + z$?

(a) -5 (b) -4 (c) 0

(d) 4 (e) 5

7. **Core Concept D.**

Write an equation or inequality that has

(a) no real solutions;
(b) infinite numbers of real solutions; and
(c) exactly one real solution.

8. **Core Concept D.**

In (a)–(f), how many solutions are there? Are they rational, real, or complex? Give a reason for your answer that does not depend on solving the equation.

(a) $(x + 3)^2 = 9$
(b) $(x - 3)^2 = 9$
(c) $-(x - 3)^2 = 9$
(d) $16 - (x - 3)^2 = 9$
(e) $9(x + 3)^2 = 0$
(f) $(x + 3)^2 = (x + 4)^2$

9. **Core Skill 1.**

One firm offers an investment plan that pays a flat rate of 10% interest each year on the original sum invested. So each dollar grows after n years to

$$(1 + \frac{10n}{100}) \text{ dollars.}$$

Another firm offers a plan that pays 5% interest each year on the previous year's balance. So each dollar grows after n years to

$$(1 + \frac{5}{100})^n \text{ dollars.}$$

Find, using a graphing calculator or a spreadsheet, when the two offers give roughly equal returns. Which is better in the long term?

10. **Equations: Core Skill 1, Core Skill 2; Functions: Core Skill 4.**

Quinn works in Chicago and in New York City. He travels by taxi in each of the two cities.

In Chicago, he pays a fixed taxi fare of \$1.90 per ride, plus \$1.60 per mile traveled.

(a) Write an equation that expresses f, Quinn's total fare for a taxi ride in Chicago, as a function of m, the number of miles traveled.

In New York City, Quinn pays a fixed taxi fare of \$1.50 per ride, plus 25 cents per $\frac{1}{10}$ mile traveled.

(b) Write an equation that expresses f, Quinn's total fare for a taxi ride in New York City, as a function of m, the number of miles traveled.

(c) On a recent trip Quinn noticed that the total number of miles traveled by taxi from the airport to the hotel was the same in each of the two cities. Before tips were added, his taxi fare to the hotel in New York City was \$12.20 more than his taxi fare to the hotel in Chicago. What was the distance from the airport to the hotel in each city? Show or explain how you got your answer.

11. **Core Skill 1, Core Skill 4.**
'Give me 8 sheep and then we will have an equal number' said one shepherd to another. 'No, you give me 8 sheep and then I will have twice as many as you' replied another shepherd. How many sheep did each shepherd have to start with?

12. **Core Skill 1, Core Skill 4.**
(*The Abbot of Canterbury's Puzzle: AD 735–804*)
One hundred bushels of corn were distributed among one hundred people in such a way that each man received three bushels, each woman received two bushels, and each child received half a bushel. Given that there were five times as many women as men, how many children were there?

13. **Core Skill 2.**
Solve the equations
(a) $\frac{24}{x-4} = 1$,
(b) $12 - 2(5 - y) = 5y$.

14. **Core Skill 2.**
Solve the following quadratic equations using factoring methods:
(a) $2x^2+x-3 = 0$ (b) $4x^2 + 6x = 0$
(c) $36x^2 - 25 = 0$ (d) $x^2+6x+9 = 0$

15. **Core Skill 2.**
Use the quadratic formula to solve $4x^2 - 2x = 5$.

16. **Core Skill 3.**
The distance d travelled after time t at a steady speed v is given by

$$d = vt.$$

(a) How long would it take to travel 150 miles at 60 miles per hour?
(b) How fast would you have to go to do it in one and a half hours?

17. **Core Skill 3.**
Solve $A = p + prt$ for p.

18. **Core Skill 3.**
The distance D that it takes a car moving at speed v to stop is given by

$$D = rv + \frac{v^2}{2a}$$

where r is the reaction time it takes the driver to hit the brakes and a is the breaking deceleration.
(a) If r is 2 seconds and $a = 5$ meters/sec^2, how far would the car travel when stopping from 50 meters/sec?
(b) Solve the given equation to find an expression for v in terms of D, r and a. If you want to stop within 100 meters, how fast can you safely go?

COMMENTS ON SOLUTION:
Part (b) goes beyond the scope of Core Skill 3, and illustrates the extension of this task into later course work.

19. **Core Skill 4.**
Solve the simultaneous equations

$$\begin{cases} 2x - y = 16 \\ 3x + 2y = 17. \end{cases}$$

20. **Core Skill 4.**
The only coins that Alexis has are dimes and quarters.
- Her coins have a total value of \$5.80.
- She has a total of 40 coins.

Which of the following systems of equations can be used to find the number of dimes, d, and the number of quarters, q, that Alexis has?

(a)

$$d + q = 5.80$$
$$40d + 40q = 5.80$$

(b)

$$d + q = 40$$
$$5.80d + 5.80q = 40$$

(c)

$$d + q = 5.80$$
$$0.10d + 0.25q = 40$$

(d)

$$d + q = 40$$
$$0.10d + 0.25q = 5.80$$

21. **Core Skill 4.**
    Solve the following system of equations:

$$\begin{cases} 2x - y - z = 7 \\ 3x + 5y + z = -10 \\ 4x - 3y + 2z = 4. \end{cases}$$

22. **Core Skill 5.**
    Solve $|x - 6| \leq 4$ and locate the solution on the number line.

23. **Core Skill 5.**
    Solve $8.5 < 3x + 2 \leq 9.7$ and locate the solution on the number line.

24. **Core Skill 1, Core Skill 5.**
    Mr. Smith uses the following formula to calculate students' final score C in his Algebra II class: $C = 0.4E + 0.6T$, where E represents the score on the final exam, and T represents the average score of all tests given during the grading period. All tests and the final exam are worth a maximum of 100 points. The minimum passing score on tests, the final exam, and the course is 60. Determine the inequalities that describe the following situation. When necessary, round scores to the nearest tenth.
    (a) Is it possible for a student to have a failing test score average (i.e., $T < 60$ points) and still pass the course?
    (b) If you answered "yes," what is the minimum test score average a student can have and still pass the course? What final exam score is needed to pass the course with a minimum test score average?
    (c) A student has a particular test score average. How can (s)he figure out the minimum final exam score needed to pass the course?

25. **Core Skill 3.**
    If oil should ever be spilled into the Columbia River Estuary, the company responsible for the spill would be liable for monetary damages according to a formula. By Washington state law, the formula in 2009 was given by:[1]

$$D = 0.508GS(A + B + C)$$

In this formula, D is the damage liability in dollars; G is the number of gallons spilled; S is a "vulnerability score" in the range from 1 to 5 that takes into account the wildlife characteristics of any given square kilometer of the estuary[2]; and A, B and C are "chemical penalty scores" in the range from 1 to 5 that take into account the toxicity, harmful mechanical properties, and longevity of the material spilled. For example, kerosene has a toxicity score $A = 1.4$, a harmful mechanical property score $B = 2.4$, and a longevity score $C = 1$.[3] Suppose that a company responsible for a kerosene spill in an area of lowest vulnerability is held liable for \$10 million. How many gallons were spilled? How many dollars per gallon was the company charged for the spill? In general, what is a formula for the number of dollars of liability per gallon of spill? What is the maximum possible liability in dollars per gallon?

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Modeling

Core Concepts

A Mathematical models involve choices and assumptions that abstract key features from situations to help us solve problems.

B Even very simple models can be useful.

Core Skills

1. Model numerical situations.
2. Model physical objects with geometric shapes.
3. Model situations with equations and inequalities.
4. Model situations with common functions.
5. Model situations using probability and statistics.
6. Interpret the results of applying a model and compare models for a particular situation.

The Modeling standard needs discussion in relation to the other standards. For example, what is the difference between a *geometry task* versus *a modeling task that uses geometry*? What is the difference between a contextualized algebra problem and a modeling problem that uses equations to describe a situation?

In these standards, a task is considered to belong more in Modeling, the more it is the case that:

- The math techniques to be used are not stated explicitly in the problem.
  - However, beginning/developmental modeling tasks can walk the student through the techniques, as a way to show their use.
- Various assumptions must be imposed by the student to apply the techniques; these assumptions are not explicitly stated in the problem; and differing sets of assumptions could all be considered reasonable.
- The task involves making a decision about something.
- The task involves an optimization of some kind.
- The context is not a pretext. While the task inevitably teaches mathematics, its primary focus is the situation or phenomenon at hand. See the figure below (after a diagram by Malcom Swann, in Muller and Burkhardt, 2006).

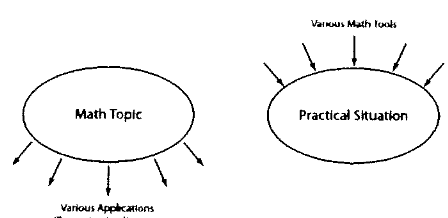

Figure 1

- The phenomenon or situation is interesting or worthwhile beyond the academic discourse of the classroom.

A list of criteria such as this one does not define a hard-and-fast rule that could be used to unambiguously classify tasks. The distinction between *geometry* and *modeling with geometry* is a heuristic one. Moreover, few, if any, of the following Modeling tasks satisfy all of the listed criteria. But each task fulfills one or more of these criteria to a great enough extent to be considered an example of Modeling.

Reference

E. Muller and H. Burkhardt, Application and Modelling for Mathematics—Overview. In W. Blum and P.L. Galbraith (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study*, Springer, 2006. http://books.google.com/books?id=XICuY-BFaHYC.

Example Tasks

1. **Core Concept A; Core Skill 2.**
   If everyone in the world went swimming in Lake Michigan, what would happen to the water level? (Would Chicago be flooded?)
2. **Core Concept A; Core Skill 2; Core Skill 7.**
   The Federal Communications Commission (FCC) needs to assign radio frequencies to seven new radio stations located on the grid in the accompanying figure. Such assignments are based on several considerations including the possibility of creating interference by assigning the same frequency to stations that are too close together. In this simplified situation it is assumed that broadcasts from two stations located within 200 miles of each other will create interference if they broadcast on the same frequency, whereas stations more than 200 miles apart can use the same frequency to broadcast without causing interference with each other. The FCC wants to determine the smallest number of frequencies that can be assigned to the six stations without creating interference.

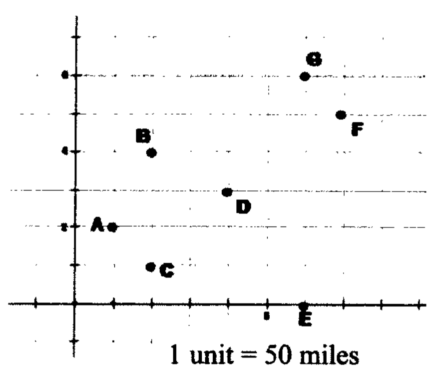

Figure 2

- Student 1 began thinking about the problem by drawing a circle of radius 200 miles centered on each radio station.
- Student 2 began thinking about the problem by drawing line segments to connect pairs of radio stations whenever the two radio stations are within 200 miles of one another.
- Student 3 began thinking about the problem by drawing line segments to connect pairs of radio stations whenever the two radio stations are more than 200 miles from one another.

Which approach seems most promising to you? Use this approach to determine the smallest number of frequencies that can be assigned to the six stations without creating interference. Justify your final answer.

3. **Core Concept A; Core Skill 2; Core Skill 3; Core Skill 5; Equations Core Concept A; Equations Core Skill 3.**

Picture a tightly rolled spool of paper. Everyday examples might include a roll of bathroom tissue or a roll of cash-register tape. See the accompanying figure.

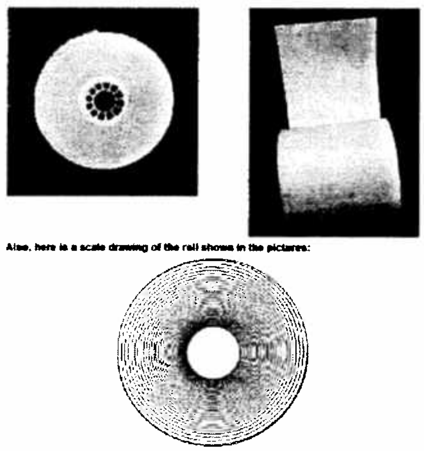

Figure 3

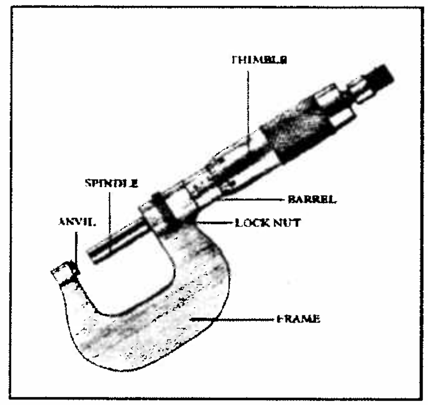

Figure 4

(a) Assuming the paper in the roll is very thin, what is the relationship between the thickness of the paper, the inner and outer diameters of the roll, and the length of the paper in the roll? Express your answer as an algebraic formula involving the four listed variables.

(b) A roll of masking tape is another example of a tightly rolled spool. In one classroom, layers of masking tape of various thicknesses were measured using a micrometer, a tool for measuring small distances (see the photo; image from http://www.design-technology.org/CDT10micrometer.htm). The table below shows the micrometer readings.

Table 1

Number of Tape Layers	0	1	2	3	4	5	6	7
Thickness (millimeters)	0.1	0.24	0.37	0.49	0.6	0.71	0.81	0.92

When you look at the first column of the table, you may wonder why zero layers of tape have a thickness of 0.1 millimeters! The reason is that these measurements were made by sticking pieces of tape to a sheet of paper (see the photo below). So the first value in the table simply represents the thickness of the piece of paper.

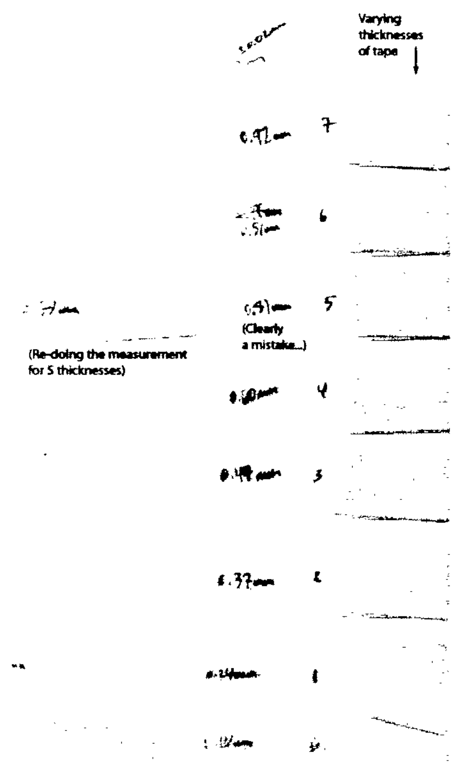

Figure 5

(i) Estimate the thickness of a single layer of tape by subtracting values in the first and second rows of Table 1.

(ii) Make a scatterplot of the data in Table 1.

(iii) Draw a straight line that you think best describes the pattern in your scatter plot.

(iv) Find the equation of this line. Explain how you determined your equation.

(v) Use your equation to estimate the thickness of a single piece of tape.

(vi) Which estimate of the thickness of a single piece of tape do you think is more accurate, the one you found in Question (i) or the one you found in Question (v)? Why?

(vii) What is your best estimate of the thickness of the paper in this experiment?

(viii) In a roll of masking tape, the inner radius is 60 millimeters and the outer radius is 80 millimeters. Using the thickness value determined in Question (v), how long is this roll of tape?

4. **Core Concept A; Core Concept B; Core Skill 1.**
   - In a country with 300 million people, about how many high school math teachers will be needed? Try to estimate a sensible answer using your own everyday knowledge about the world. Write an explanation of your answer, stating any assumptions you make.
   - Likewise, estimate the number of people born each day on planet earth.
   - Likewise, estimate the percentage of Americans who are pregnant at any given time. Also estimate the percentage of elephants who are pregnant at any given time.

5. **Core Concept B; Core Skill 1.**

Suppose you wrote check #556 on November 5, 1995, and check #953 on September 26, 1997. What is a good guess for when you wrote check #678?

6. **Core Skill 1.**

The accompanying figure shows a conversation that occurred during a cross-country road trip.

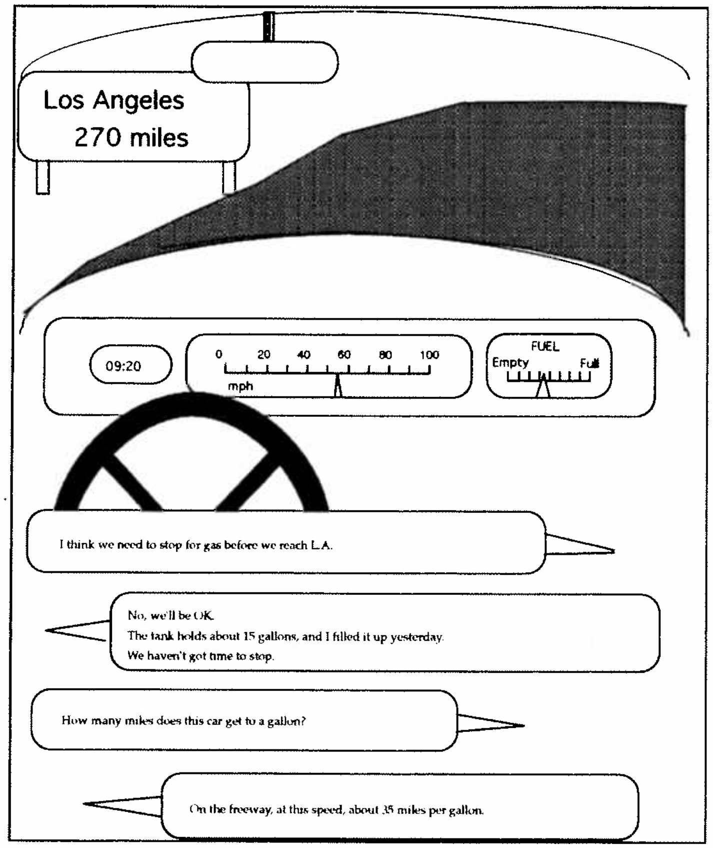

Figure 6

(a) Do they have to stop for gas? Explain your reasoning.
(b) Suppose they decide to stop for 30 minutes. At what time will they reach Los Angeles?

7. **Core Skill 1; Core Skill 2; Core Skill 3.**

A team of haymakers was assigned the task of scything two meadows, one twice the size of the other. The team worked half a day on the larger meadow. Then it split into two equal groups. The first group remained in the larger meadow and finished it by evening. The second group scythed the smaller meadow, but by evening there still remained a portion to do. This portion was scythed the next day by one haymaker in a single day's work. How many haymakers were there in the team?

8. **Core Skill 1; Core Skill 3.**

A car does y miles a year, averaging m miles per gallon.

(a) Write a formula for g, the number of gallons used in a year.

(b) If the average price of gas is $\$p$ per gallon, write a formula for the total cost $\$C$.

(c) You are thinking of changing your 15 mpg gas guzzler for a 40 mpg car. If you drive 20,000 miles each year, how much money would you save on gas at \$3 per gallon? How much is this savings per week?

9. **Core Skill 1; Core Skill 3.**

Funds totaling \$191,000 are designated for four schools. The distribution of the funds is to be in proportion to the number of students in each school. Student populations of the four schools are: School A, 386; School B, 1691; School C, 2109; School D, 817.

(a) Figure out how much money each school gets.

(b) Draw some sort of diagram to scale that helps show visually how the money is divided up among the four schools.

(c) Find general formulas for deciding how much money each of the four schools gets in terms of the populations of the schools and the total amount of money to be distributed.

(d) Add your formulas algebraically. What is the result? Does this make sense?

10. **Core Skill 2.**

Ann is moving to Gridville. She's looking for a house to live in. The location of her house-to-be should satisfy the following conditions:

- Ann wants to live close to the office where she works, so the distance from her house to the office should not be more than 2.5 km.
- Because of the terrible smell of the pet food factory, the distance from her house to the factory must be at least 3 km.

(a) Both conditions mention distance. Is distance measured in the same way in both situations? If yes, why? If no, why not?

(b) On the grid provided, graph the area that meets both conditions.

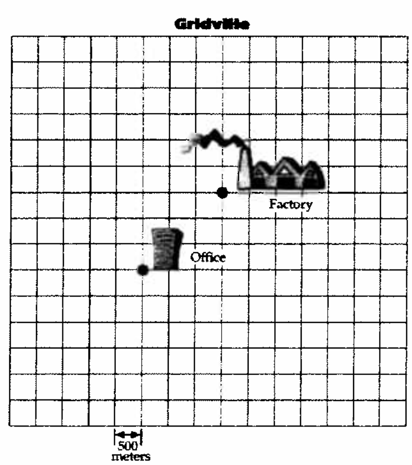

Figure 7

11. **Core Skill 2.**

A poster manufacturing company is considering different ways of making and shipping its posters. The figure below shows two kinds of tubes for posters: square tubes and hexagonal tubes.

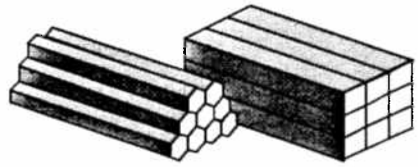

Figure 8

The square tubes are 7.5 × 7.5 cm, with a length of 75 cm. The hexagonal tubes have the same length of 75 cm but they have a diameter of 9 cm (see figure below).

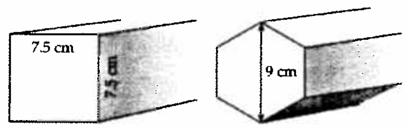

Figure 9

From the customer's perspective, a more tightly rolled poster is harder to flatten and hang on a wall. So a tube that allows the poster to be more loosely rolled is more desirable than a tube that requires the poster to be more tightly rolled. How tightly a poster is rolled is based on the amount of space measured by the largest circle that fits into the tube. See the figure below.

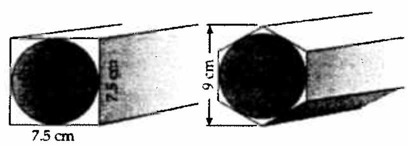

Figure 10

(a) Calculate the area of the circle in the square tube and the area of the circle in the hexagonal tube. Explain your answers.
(b) Calculate the efficiency of both tubes in terms of percentage of space used by the circular poster roll and its interior. Explain your answers.
(c) Based on the information you have now, in which tube do you think the company should pack posters? Why?
(d) The individual hexagonal tubes are packed in a 20 × 20 × 75 cm box (see the Figure below). Calculate the efficiency in terms of the percentage of the box's front that is used by the hexagons.

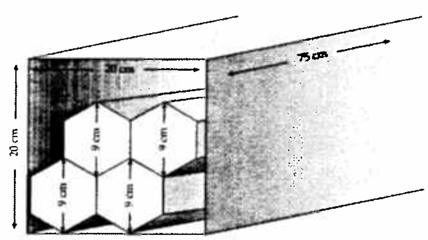

Figure 11

(e) How many of the square tubes will fit in the 20 × 20 × 75 cm box? (You may want to make a sketch to help you decide.)
(f) Calculate the efficiency in terms of the percentage of the box's front that is used by the squares.
(g) Based on the information you have now, in which tube do you think the company should stack posters? Why?
(h) The poster company is considering a larger box for shipping: 45 × 30 × 75 cm.
  (i) How many hexagonal tubes fit in the box?
  (ii) How many square tubes fit in the box?
  (iii) For both the hexagonal tubes and square tubes, find the efficiency in terms of the percentage of the box's front that is used by the hexagons or squares.
(i) Based on the information you have now, in which tube do you think the company should stack posters? Why?

12. **Core Skill 2.**
Based on enrollment predictions school officials have decided that, for the next school year, the classroom trailers will be moved to the current practice football field and a new practice field will be located behind the school parking lot. Including end zones, the practice field will be 120 yards by 53 yards in order to closely approximate a standard field. However, the owner of a local nursery has donated enough grass seed to plant 81,000 square feet. Since they have more than enough grass seed for the practice field, school officials would like to plant a uniform border around the field. What are the dimensions of the 81,000 square feet rectangular area that should be planted for the practice field and uniform border?

13. **Core Skill 2.**
The accompanying figure shows the plan of a city. All curved and straight lines are streets.

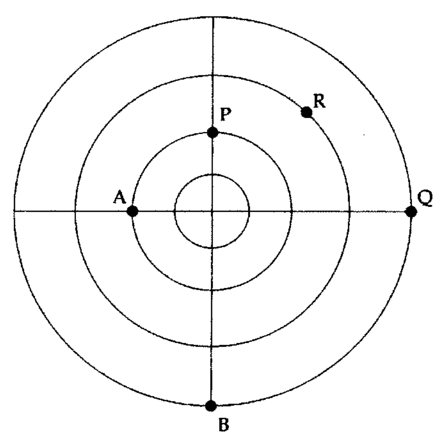

Figure 12

(a) On the figure, draw the shortest route from P to Q and the shortest route from P to R.
(b) On the figure, draw three points that are approximately equidistant from A and B (using streets).
(c) Compare and constrast this type of distance with other types of distance you have studied.

14. **Core Skill 2.**
The accompanying figure is a diagram of a miniature golf hole. Players must start their ball from one of the three tee positions. A wall separates the tees from the hole.

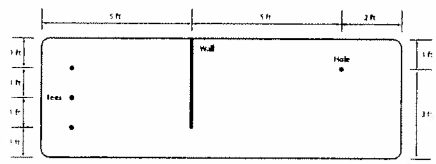

Figure 13

At which tee should the ball be placed to create the shortest "hole in one" path? What is this shortest distance? Explain your reasoning.

15. **Core Skill 2; Core Skill 3.**

All states have building codes. Many such codes can be interpreted as mathematical inequalities, since they establish limits on what can be done.

Most states have codes related to staircase construction (see the photo).

Figure 14

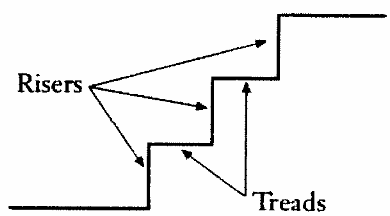

Figure 15

The most basic dimensions of a set of stairs are *riser height* and *tread depth.* A *riser* is the vertical front of a stair. The surface that you step on is called a *tread.* (See the diagram.)

(a) The Massachusetts State Building Code includes this statement:

> Maximum riser height shall be seven inches (178 mm) and minimum riser height shall be four inches (102 mm).

This statement contains two requirements for riser height. Write these requirements in two ways: as a pair of simple inequalities, and also as a compound inequality.

(b) Here is another statement from the Massachusetts State Building Code:

> Minimum tread depth shall be 11 inches (279 mm), measured horizontally between the vertical planes of the foremost projection of adjacent treads at a right angle to the tread's leading edge.

Write this requirement as an inequality.

(c) In addition, most staircases conform to this design guideline:

The depth of a tread plus two times the height of a riser should have a total value of from 24 to 25 inches.

Write this guideline using inequalities.

(d) Label a suitable coordinate grid. Graph the solution to the system of inequalities regarding staircase riser height and tread depth.

(e) Choose an appropriate point to check your answer.

16. **Core Skill 2; Core Skill 3; Core Skill 4; Quantity; Equations, Core Skill 3.**

A manufacturer wants to design a cylindrical soda can that will hold 500 milliliters of soda. The manufacturer's research has determined that an optimal can radius is between $3\frac{1}{4}$ and 4 centimeters. What is the corresponding range of possible height measurements for the can? Explain your reasoning.

17. **Core Skill 3.**

A group of art students has formed an Arts Collective to organize an arts and crafts exhibition (see photo).

Figure 16

In order to publicize the event, they intend to use two methods. They can print one-page flyers for distribution at schools and malls. The flyers will cost only eight cents each to produce. Another method is to mail postcards about the event to selected people. This option is more expensive: twelve cents for each card plus eighteen cents per card for bulk postage. Postcards are expected to be more effective than flyers. But in order to get the twelve cent rate, they must order at least 1000 cards. The total budget for publicity is \$1200.

The students must decide how many flyers to print and how many cards to order. But there are upper and lower limits on the number of each.

(a) What is the lower limit on the number of cards they will order if they want the quantity discount?

(b) Let C represent the number of cards ordered. Write the lower limit as a mathematical inequality.
(c) What is the lower limit on the number of flyers that can be produced?
(d) Let F represent the number of flyers produced. Write the lower limit as an inequality.
(e) One statement in the problem description implies an upper limit on a quantity. Explain.
(f) Write a sentence that describes this upper limit in more detail. Include the phrases "money spent on flyers" and "money spent on cards."
(g) Rewrite your answer to Question (f) as a mathematical inequality in the variables C and F.
(h) Your answers to Questions (b), (d), and (g) taken together form a system of inequalities in C and F. The solution of this system represents the range of publicity options available to the Arts Collective. The solution can be visualized with a graph. Should the boundaries of the solution region be solid lines or dashed lines? Explain.
(i) The inequalities in Questions (b) and (d) should each contain only one variable, so they are probably easier to graph. Label the axes of a two-dimensional coordinate grid with the variables C and F. Then graph the inequalities from Questions (b) and (d).
(j) Now consider the budget limit. If no flyers are produced, how many cards can be bought and mailed?
(k) If the minimum of 1000 cards is used, how many flyers can be made?
(l) Your answers to Questions (j) and (k) can each be represented by a point on the graph. Use these points to graph the boundary that represents the budget limit. Then shade only the region that satisfies all three inequalities.
(m) Check your answer. Choose any point that is inside the shaded region, and verify that its coordinates satisfy all three inequalities.

18. **Core Skill 3.**

A coffee shop sells several kinds of coffee. The shop also uses some of its coffees to make its own custom blends. Coffee A sells for \$6 a pound. Coffee B sells for \$10 a pound. The shop's manager wants to create a blend of the two types that sells for \$7 a pound. The manager wants to make 10 pounds of this blend. How many pounds of each type of coffee should be in the 10 pounds of blend?

19. **Core Skill 3.**

When summer approaches, Desmond and Farid look forward to baseball games, movies, and weekends at the local amusement park. Because all of these things cost money, Desmond and Farid decide to start a lawn service to earn money.

Desmond can devote 10 hours a week to this new venture, while Farid can devote 4 hours. The boys realize it takes less time to trim the edge than to mow, so they decide that Desmond can do all the mowing and Farid can do the edging. After surveying the neighborhood, they determine that their clients will fall into one of two categories: standard-sized interior lots or large corner lots.

From working on their own yards the boys know it will take about an hour to mow and half an hour to edge a standard-sized yard. A larger yard will take 45 minutes to edge and 2 hours to mow.

Based on research he has done in his neighborhood, Desmond wants to set prices at \$20 per large yard and \$15 for each standard-sized yard. Each price is the total price that includes edging and mowing.

On the one hand, the boys might decide to do more large yards, each of which brings more money. Or they might decide to do more standard yards, each of which takes less time. It's not obvious which approach will earn them the most money. The goal of this problem is to find the best approach.

(a) Use the table below to organize the information in the situation:
(b) Using the information in the table to assist you, write a system of inequalities to represent all the constraints that define the boys' problem situation.
(c) Farid and Desmond's goal is to maximize their earnings from the lawn service. Find a formula that expresses their weekly earnings P in terms of x and y.
(d) Graph the system of inequalities representing the constraints and shade the feasible region—the region representing all of the values of x and y that obey all of the constraints.

	x (# of standard yards)	y (# of large yards)	Constraint
Number of hours mowing			
Number of hours edging			

Table 2: Desmond and Farid's situation

(e) What are the vertices of your feasible region? Explain how you found them.

(f) Use the vertices (from Question (e)) and the earnings formula to find the maximum Farid and Desmond can earn in one week with the given constraints.

(g) How many and which size yards will Farid and Desmond need to mow and edge to maximize their earnings?

20. **Core Skill 3.**

This problem is difficult and would merit an extended treatment in class; it is not meant to be one in a series of routine "word problems" on a worksheet.

Two boats (boat 1 and boat 2) start off from opposite sides of a lake, each heading for the starting point of the other. They pass each other 800 yards from the starting point of boat 1. They continue to the starting point of the other boat, turn around, and return. This time, they pass each other 300 yards from the (original) starting point of boat 2. How wide is the lake? (Each boat travels at a fixed speed. Ignore turnaround times.)

21. **Core Skill 3.**

An academic team is going to a state mathematics competition. There are 30 people going on the trip. There are 5 people who can drive and 2 types of vehicles, vans and cars. A van seats 8 people, and a car seats 4 people, including drivers. How many vans and cars does the team need for the trip? Is more than one option available? Explain your reasoning.

22. **Core Skill 4.**

Clara purchased a used car for \$8400. She estimates that each year she owns the car it will depreciate (lose value) by 12% of its value the previous year.

(a) According to Clara's assumption, the car's value after 1 year from the purchase date will be \$7392. Check this yourself.

(b) What will be the car's value 2, 3, and 4 years from the date of purchase?

(c) Clara plans to keep the car until its value reaches \$4000. By Clara's assumption, what is the minimum number of years from the date of purchase that the car's value will be less than \$4000? Show your work.

23. **Core Skill 4.**

If it is never refreshed, the water in a swimming pool becomes polluted after some time. Urea that comes into the water via body secretions is one of the substances that pollute swimming pools.

In this problem, presume an increase in the amount of urea of 500 g per day, caused by 1000 swimmers a day. Suppose the amount of urea present in the water is represented as u.

(a) Find a recursive formula for the quantity of urea in the water (per day).

(b) When there are 4000 g of urea in the water, the swimming pool will be closed to the public because the water has to be cleaned. The cleaning equipment can remove 10% of the amount of urea present in one day, so the first day, 400 g of urea can be removed. The second day, 10% of the remaining amount of urea will be removed, and so on.

(i) What recursive formula describes this process of cleaning?

(ii) How many days does it take before 50% of the 4000 g of urea is removed from the water?

(c) A better method of keeping the amount of urea under control is to let the people swim during the day and to refresh the water during the night. Starting with a clean pool (no urea in the water), swimmers will add 500 g of urea to the water every day. Every night, 10% of the urea in the pool will be removed.

(i) Calculate the amount of urea in the pool at the beginning of day 2 (after one day of swimming and one night of refreshing).

(ii) Calculate the amount of urea at the beginning of day 3.

(iii) Find a recursive formula for the amount of urea at the beginning of a day.

(iv) Will the amount of urea continue to increase without bound day after day, or will the amount of urea stabilize at some point? Explain how you found this.

24. **Core Skill 4.**

At the You're Toast, Dude! toaster company, the weekly cost to run the factory is \$1400 and the cost of producing each toaster is an additional \$4 per toaster.

(a) Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.

(b) What is the total cost of producing 100 toasters in one week?

(c) If you produce 100 toasters in one week, what is the total production cost per toaster?

(d) Will the total production cost per toaster always be the same? Justify your answer.

(e) Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.

(f) Answer the following questions:

(i) What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?

(ii) What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.

(iii) How many toasters must be produced to have a total production cost per toaster of \$8?

25. **Core Skill 4.**

A recommended adult dosage of the cold medication NoMoreFlu is 16 mL. NoMoreFlu causes drowsiness when there are more than 4 mL in one's system, in which case it is unsafe to drive, operate heavy machinery, etc. The manufacturer wants to print a warning label telling people how long they should wait after taking NoMoreFlu for the drowsiness to pass. The typical metabolic rate is such that one quarter of the NoMoreFlu is lost every four hours.

(a) If a person takes the full dosage, how long should adults wait after taking NoMoreFlu to ensure that there will be

(i) Less than 4 mL of NoMoreFlu in their system?

(ii) Less than 1 mL in their system?

(iii) Less than 0.1 mL in their system?

(b) What do you think the warning label should say? Design the label and explain the thinking behind your design.

26. **Core Skill 4; Core Skill 7.**

Among the many species that have been endangered at one time or another is the desert bighorn sheep. The desert bighorn sheep are sensitive to human-induced problems in the environment and their numbers are therefore a good indicator of land health.

It is estimated that in the 1600s, there were about 1.75 million bighorn sheep in North America. By 1960, the bighorn sheep population in North America had dropped to about 17,000. There appears to have been a similar decline in west Texas, where wildlife biologists have data showing that in 1880, there were around 1,500 bighorn sheep in west Texas and by 1955, the population had dwindled to 25 in that area. Efforts to reintroduce desert bighorn sheep in west Texas began around 1957 and by 1993, there were about 400 desert bighorn sheep in west Texas roaming free or in captivity.

(a) Assume that the annual percentage decrease in the bighorn sheep population in west Texas was fairly constant from 1880 to 1955. Model the bighorn sheep population for any year in this range with an exponential function of the form $P = a \cdot b^t$, where t is the number of years since 1880, a is the population of the bighorn sheep in west Texas in 1880, b is the annual rate of retention, and P is the annual bighorn sheep population in west Texas for the given year.

(b) Assume that, starting in 1957 when reintroduction began, the annual percentage increase in the bighorn sheep population in west Texas was fairly constant. Model the bighorn sheep population for any year since 1957 with the exponential function, $P = a \cdot b^t$, where t is the number of years since 1957, b is the annual rate of growth, P is the annual bighorn sheep population in west Texas for the given year, and a is the bighorn sheep population in 1957.

(c) Describe the mathematical domain for the two functions in Questions (a) and (b). Describe reasonable domain values for the actual situations modeled by these functions. Explain any differences between the two types of domains (mathematical and situational).

(d) From 1880 to 1955, by what percentage was the population decreasing annually? From 1957 to 1993, by what percentage was the population increasing annually?

(e) By what year had the sheep population dropped to 750 or fewer? Use numerical, graphical, or algebraic methods and explain your reasoning.

(f) If the reintroduction program continues, in what year will the bighorn sheep population again be at least 750? Use numerical, graphical, or algebraic methods and explain your reasoning.

(g) In 2001, it was reported that there were 500 bighorn sheep in west Texas. Is this number consistent with the number predicted by the exponential model for reintroduction? Why or why not?

27. **Core Skill 5.**

Our school has to select a girl for the long jump at the regional championship. Three girls are in contention. We have a school jump-off. Their results, in meters, are given in the accompanying table.

Elsa	Miki	Aisha
3.25	3.55	3.67
3.95	3.88	3.78
4.28	3.61	3.92
2.95	3.97	3.62
3.66	3.75	3.85
3.81	3.59	3.73

Table 3: Data from the jump-off; distances are given in meters.

Hans says, "Aisha has the longest average. She should go to the championship."
Do you think Hans is right? Explain your reasoning.

28. **Core Skill 5.**

The figure below shows two graphs: one showing the amount of water consumed in the United States from 1950 through 1980, and another graph showing the number of people living in the U.S. during those years.

T is the total amount of water consumed and B is the population. The vertical scale on the left side of the graph belongs to the graph of T. On the right side you see the vertical scale that belongs to the graph of B.

So, in 1970, the total water consumption was about 1340 billion liters per day and the population size was about 200 million people.

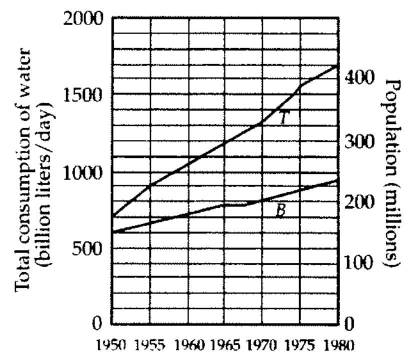

Figure 17

(a) Complete the table below with data from the graphs.

Year	1950	1955	1960	1965	1970	1975	1980
T (billion liters per day)	700				1340		
B (millions)	145				200		

(b) Use the data from the table to complete the graph below.

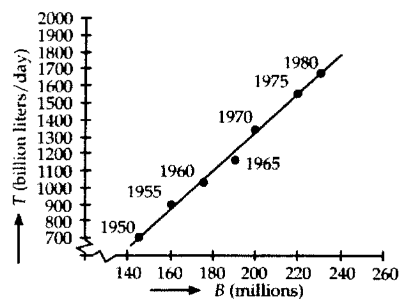

Figure 18

(c) Draw a straight line that you think best describes the pattern in your scatter plot.

(d) Find the equation of this line. Explain how you determined your equation.

(e) In 1980, researchers predicted that every five years the total consumption of water would increase by 110 to 200 billion liters per day.

(i) If that prediction is correct, what will be the minimum total consumption of water in the year 2000? Explain your answer.

(ii) What will be the maximum total consumption of water in the year 2000? Show how you got your answer.

(f) Use the experts' prediction for water-consumption increase stated in Question (e) to write equations for predicting the minimum and maximum total water consumption per day in any given year.

(g) Researchers don't expect that the increase in water consumption will continue forever. They predict that 5000 billion liters per day will be the maximum amount of water available. Between which years will people consume 5000 billion liters of water per day, according to researchers? Explain.

29. **Core Skill 5; Core Skill 6.**

The data in the accompanying table shows the annual median earnings for female and male workers in the United States from 1984 to 2004.

Year	Women's median earnings (in dollars)	Men's median (in dollars)
1984	8675	17026
1985	9328	17779
1986	10016	18782
1987	10619	19818
1988	11096	20612
1989	11736	21376
1990	12250	21522
1991	12884	21857
1992	13527	21903
1993	13896	22443
1994	14323	23656
1995	15322	25018
1996	16028	25785
1997	16716	26843
1998	17716	28755
1999	18440	30079
2000	20267	30951
2001	20851	31364
2002	21429	31647
2003	22004	32048
2004	22256	32483

(a) Create two scatter plots, one for women's median earnings over time and one for men's median earnings over time. Describe two things you notice about the scatter plots.

(b) Terry and Tomás are trying to decide what type of model will most accurately represent the data. Terry thinks that a linear model might be most appropriate for each scatter plot. Help Terry find reasonable linear function rules for each scatter plot. Explain how you found these.

(c) Using the linear models, will women's annual median earnings ever equal those of men? Why or why not?

(d) Tomás thinks that an exponential model might be most appropriate for each scatter plot. Help Tomás find reasonable exponential function rules for each scatter plot. Explain how you found these.

(e) Using the exponential models, will women's annual median earnings ever equal those of men? Why or why not?

(f) If you answered yes to either Question (c) or question (e), use that model to determine the first year women will have higher median earnings than men. Explain how you found your answer.

(g) For each year listed in the table, find the ratio of women's to men's annual median earnings expressed as a percentage. Use the data to create a scatter plot of percentage versus year. Based on this graph, do you think women's annual median earnings will ever equal those of men? Why or why not? Considering the results of the scatter plot in Question (g) above, do you think the linear model or exponential model makes more sense? Why?

30. **Core Skill 5; Core Skill 7.**

Karnataka is a state in southwest India. The accompanying table is agricultural data on fertilizer use and grain crop yield in Karnataka. Fertilizer is measured in 100,000 tons. Crop yield is measured in 10 kilograms per hectare.

Year	Fertilizer ($\times 10^5$ tons)	Yield ($\times$ 10 kg/ha)
1956	0.06	52.3
1966	0.38	45.8
1970	1.32	19.1
1974	1.95	88.9
1975	2.32	82.9
1976	2.03	93.9
1977	2.08	79.2
1978	2.9	97.9
1979	3.72	101.7
1980	3.59	102.9
1981	3.54	91.9
1982	3.87	102.2
1983	3.83	85.8
1984	4.87	102.1
1985	5.9	95.4
1986	5.55	81.19
1987	5.66	97.03
1988	5.58	83.57
1989	5.75	93.5

Throughout the years over which these data were gathered, the amount of land in cultivation remained fairly constant.

(a) Find a mathematical function that you think does a good job of modeling the relationship between fertilizer use and grain crop yield. Explain how you did it.
(b) Use the function you have chosen to predict the yield if fertilizer use is 500,000 tons.
(c) How comfortable are you with the prediction you made in Question (b)? Explain.
(d) What advice can you offer the government of Karnataka about fertilizer use? Explain.

31. **Core Skill 6.**

"My sixty-year old mother, who lives in New York, gets frightened by newspapers. One day she is afraid of being a victim of crime, the next she is frightened of being killed in a road accident, then it's terrorists, and so on."

(a) Using reliable websites with national statistics...
   (i) Estimate the chances of my mother being a victim of the above events during the next year.
   (ii) Compare the likelihood of these events with the probability that women of her age will die during the coming year.

(b) Why do you suppose people fear such unrealistic dangers?

32. **Core Skill 6.**

In the general population, about 1 baby in 8,000 dies in an unexplained "crib death." The cause or causes are at present unknown. Three babies in one familiy have died. The mother is on trial, and you are on the jury. An expert witness says: "One crib death is a family tragedy; two is deeply suspicious; three is murder. The odds of even two deaths in one family are 64 million to 1."

Think about the reasoning underlying this testimony. Check the expert's math, of course, but also explain what assumptions are being made. Would the expert's testimony lead you to decide that a murder had taken place?

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Statistics

Core Concepts

A Statistical methods take variability into account to support making informed decisions based on quantitative studies designed to answer specific questions.

B Visual displays and summary statistics condense the information in data sets into usable knowledge.

C Randomness is the foundation for using statistics to draw conclusions when testing a claim or estimating plausible values for a population characteristic.

D The design of an experiment or sample survey is of critical importance to analyzing the data and drawing conclusions.

Core Skills

1. Formulate questions that can be addressed with data. Identify the relevant data, collect and organize it to respond to the question.
2. Use appropriate displays and summary statistics for data.
3. Interpret data displays and summaries critically; draw conclusions and develop recommendations.
4. Draw statistical conclusions involving population means or proportions using sample data.
5. Evaluate reports based on data.

Example Tasks

1. **Core Concept A, Core Skill 5.**
   An American Automobile Association report includes the graph shown below.

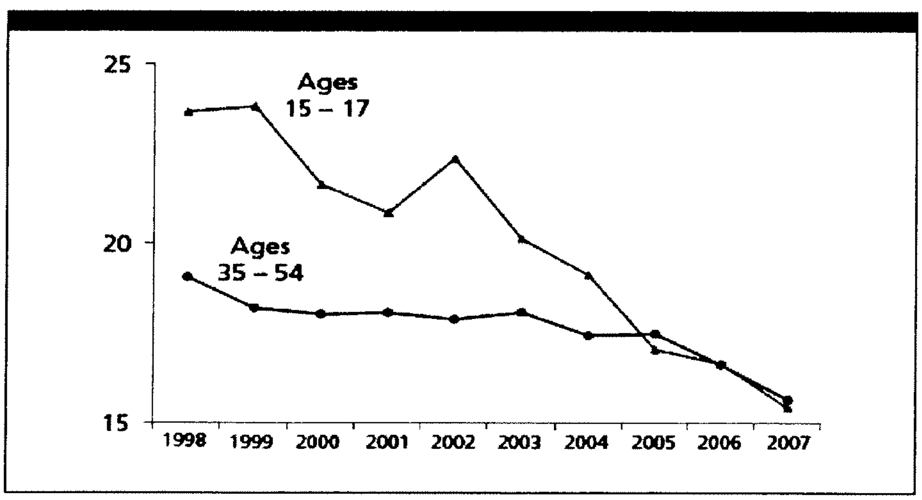

Figure 1: Number of drivers involved in fatal crashes per 100,000 population, 1998–2007.

(a) Provide a written version of the information provided by this graph.
(b) The graph seems to imply that teen-aged drivers are becoming just as good as adult drivers in the 35-54 age group. Is that necessarily true? What other variables may have to be taken into account in comparing drivers for the two age groups?

(c) One of the variables that may have been listed in part 2 is the number of miles driven. How do you think rates reported in "number of drivers involved in fatal crashes per 100,000 miles driven" would differ from the rates on the graph?

2. **Core Concept B, Core Skill 2.**

Suppose the summary statistics for the number of inches of rainfall in Northampton, MA for the past 117 years, beginning in 1877, are given below.

N	Mean	Median	Min	Max	Q1	Q3
117	42.8	41.4	22.5	68.0	37.3	48.4

(a) Sketch a boxplot for these data and describe the key features of the data distribution.
(b) The news media reported that in a particular year, there were only 38 inches of rainfall. Is it appropriate for the news media to use the word only in the statement? In other words, is 38 inches an unusually small amount of rainfall for Northampton? Explain your answer.

3. **Core Concept B, Core Skill 2.**

Students in Ms. Garth's Algebra II class wanted to see if there are correlations between test scores and height and between test scores and time spent watching television. Before the students began collecting data, Ms. Garth asked them to predict what the data would reveal. Answer the following questions that Ms. Garth asked her class.

- Do you think students' heights will be correlated to their test grades? If you think a correlation will be found, will it be a positive or negative correlation? Will it be a strong or weak correlation?
- Do you think the average number of hours students watch television per week will be correlated to their test grades? If you think a correlation will be found, will it be a positive or negative correlation? Will it be a strong or weak correlation?

The students then created a table in which they recorded each student's height, average number of hours per week spent watching television (measured over a four-week period), and scores on two tests. Use the actual data collected by the students in Ms. Garth's class, as shown in the table below, to answer the following questions.

Student	Height (in inches)	TV hrs/week (average)	Test1	Test2
1	60	30	60	70
2	65	12	80	85
3	51	30	65	75
4	76	20	85	85
5	66	10	100	100
6	72	20	78	88
7	59	15	75	85
8	58	12	95	90
9	70	15	75	90
10	67	11	90	90
11	65	16	90	95
12	71	20	80	85
13	58	19	75	85

(a) Which pairs of variables seem to have a positive correlation? Explain.
(b) Which pairs of variables seem to have a negative correlation? Explain.
(c) Which pairs of variables seem to have no correlation? Explain.
(d) For each pair of variables listed below, create a scatter plot with the first variable shown on the y-axis and the second variable on the x-axis. Are the two variables correlated positively, correlated negatively, or not correlated? Determine whether each scatter plot suggests a linear trend. Do the plots confirm your decisions in parts (a)-(c) above?
  - Score on test 1 versus hours watching television
  - Height versus hours watching television
  - Score on test 1 versus score on test 2
  - Hours watching television versus score on test 2
  - Height versus score on test 1
(e) Using the statistical functions of your graphing calculator, determine a line of best fit for each scatter plot that suggests a linear trend.

4. **Core Concept B, Core Skill 2, Core Skill 3.**
The table below[1] shows the reported high school graduation rates (percent of 9th grade cohorts who actually graduate), the per-pupil expenditure, PPE (in hundreds of dollars), and the percent of total taxable resources the state spends on education, PTR, for a sample of 15 states.

State	Graduation Rate	PTR	PPE
Alabama	59.0	3.4	79.24
Alaska	65.1	3.5	85.62
California	70.7	3.3	70.81
Colorado	74.6	3.0	79.39
Florida	60.5	3.0	75.39
Kansas	74.4	3.8	88.62
Kentucky	70.0	3.4	79.78
Louisiana	61.4	2.9	85.82
Michigan	69.1	4.5	91.97
Montana	76.2	3.7	89.51
Nevada	54.0	2.8	71.41
New Hampshire	76.0	3.9	93.23
New Mexico	60.1	3.7	84.31
North Dakota	79.4	3.1	91.81
Wisconsin	77.3	4.1	101.99

(a) Use technology to plot Graduation Rate versus PPE on a scatter plot that allows prediction of Graduation Rate from PPE and describe the shape of the distribution.
(b) If a linear shape is apparent, use statistical software or calculator capability to fit a least squares line to summarize the nature of that trend.

[1]Source: EPE Research Center, Education Week

(c) Interpret the slope of the least squares line found in part 2 in the context of the data.
(d) Is the relationship between Graduation Rate and PPE stronger than the one between Graduation Rate and PTR? Explain.

5. **Core Concept B, Core Skill 3.**
How did housing prices change during the turbulent economic times around 2008? One way to answer the question is to compare median prices for a variety of locations over a short time span. The data display below shows box plots of the median sales price (in thousands) of existing single-family houses for a sample of 25 metropolitan areas around the United States at mid-year 2005 and 2008.

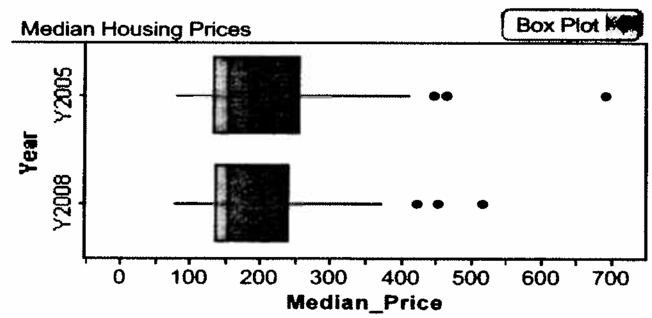

Figure 2

(a) Describe the key features of the shift in prices from 2005 to 2008.
(b) The medians of the distributions appear to differ by only a very small amount. Do you think the means would differ by a larger amount? Explain your reasoning.
(c) Why do you think the median is often used for housing prices rather than the mean?
(d) The accompanying scatter plot shows the median housing prices for 25 metropolitan areas in 2005 paired with the prices in 2008.
   (i) Describe the shape, trend and strength of the relationship between these two variables.
   (ii) Sketch the line y=x on this plot. What does the plot show with regard to the nature of the change in median prices?

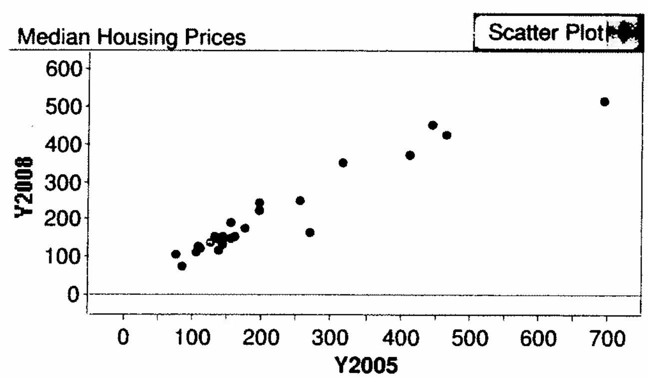

Figure 3

6. **Core Concept B, Core Skill 3.**
The histogram below shows the mean annual temperatures in Pasadena for the years 1951 to 2000.

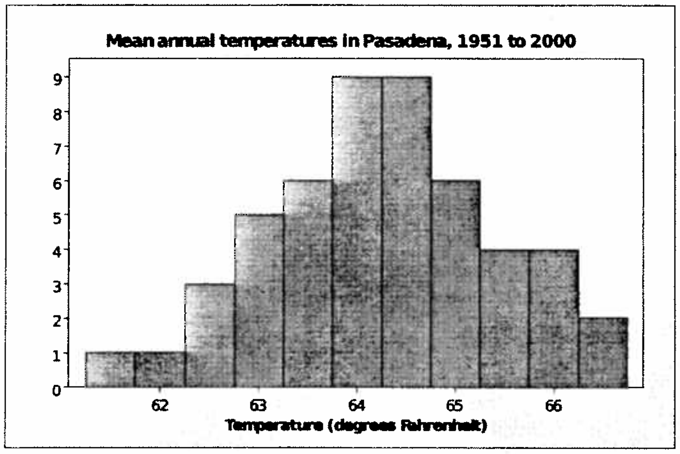

Figure 4

(a) Describe the distribution of mean annual temperatures at Pasadena.
(b) The scatter plot below shows the mean annual temperatures in Pasadena for the years 1951 to 2000. What do we learn from the scatter plot that is not obvious from the histogram.

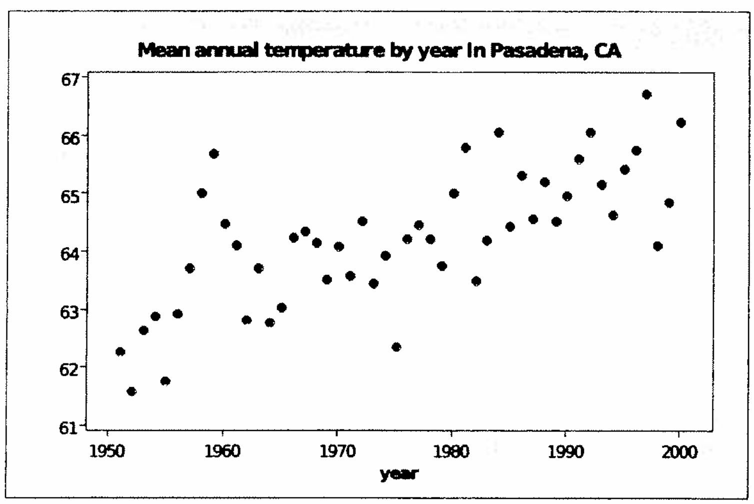

Figure 5

(c) What do we learn from the histogram that is not obvious from the scatter plot.

7. **Core Concept B, Core Skill 3.**

To determine the amount of sugar in a typical serving of breakfast cereal, a student randomly selected 60 boxes of different types of cereal from the shelves of a large grocery store. The student noticed that the side panels of some of the cereal boxes showed sugar content based on one-cup servings, while others showed sugar content based on three-quarter-cup servings. Many of the cereal boxes with side panels that showed three-quarter-cup servings were ones that appealed to young children, and the student wondered whether there might be some difference in the sugar content of the cereals that showed different-size servings on their side panels.

To investigate the question, the data were separated into two groups. One group consisted of 29 cereals that showed one-cup serving sizes; the other group consisted of 31 cereals that showed three-quarter-cup serving sizes. The boxplots shown below display sugar content (in grams) per serving of the cereals for each of the two serving sizes.

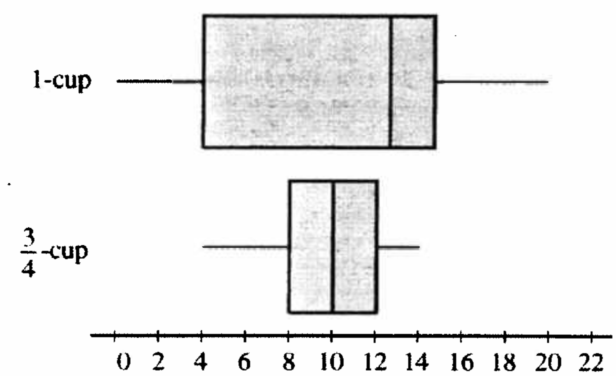

Figure 6

(a) Compare the distributions of sugar content per serving for the two serving sizes of cereals.

After analyzing the boxplots above, the student decided that instead of a comparison of sugar content per recommended serving, it might be more appropriate to compare sugar content for equal-size servings. To compare the amount of sugar in serving sizes of one cup each, the amount of sugar in each of the cereals showing three-quarter-cup servings on their side panels was multiplied by $4/3$. The bottom boxplot shown below displays sugar content (in grams) per cup for those cereals that showed a serving size of three-quarter-cup on their side panels.

(b) What new information about sugar content do the boxplots below provide?

(c) Based on the boxplots below, how would you expect the mean amounts of sugar per cup to compare for the different recommended serving sizes? Explain your reasoning.

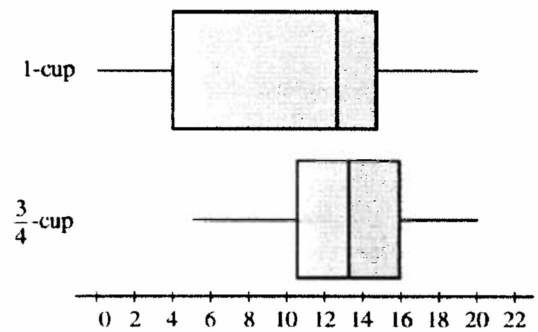

Figure 7

8. **Core Concept B, Core Skill 3.**

Two pain relievers, A and B, are being compared for relief of post-surgical pain. Twenty different strengths (doses in milligrams) of each drug were tested. Eight hundred post surgical patients were randomly divided into 40 different groups. Twenty groups were given drug A. Each group was given a different strength. Similarly, the other twenty groups were given different strengths of drug B. Drug strengths used ranged from 210 to 400 milligrams. Thirty minutes after receiving the drug, each patient was asked to describe his or her pain relief on a scale of 0 (no decrease in pain) to 100 (pain totally gone).

The strength of the drug, given in milligrams, and average pain rating for each group are shown in the scatter plot below. Drug A is indicated with A's and drug B with B's.

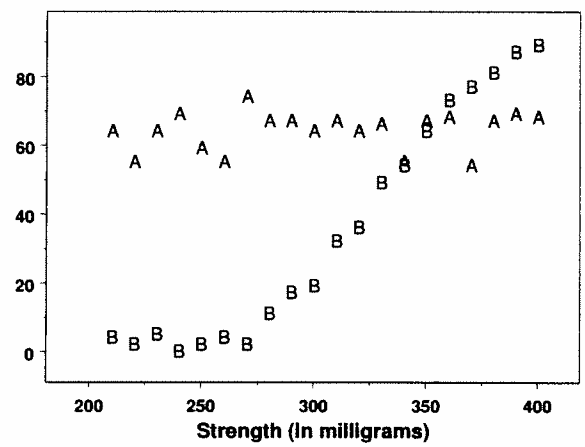

Figure 8

(a) Based on the scatter plot, carefully describe the effect of drug A and how it is related to strength in milligrams.

(b) Based on the scatter plot, carefully describe the effect of drug B and how it is related to strength in milligrams.

(c) Which drug would you give and at what strength, if the goal is to get pain relief of at least 50 at the lowest possible strength? Justify your answer based on the scatter plot.

9. **Core Concept B, Core Skill 3.**

SAT mathematics scores for a particular year are approximately normally distributed with a mean of 510 and a standard deviation of 100.

(a) What is the probability that a randomly selected score is greater than 610? Greater than 710? Between 410 and 710?

(b) If a student is known to score 750, what is the student's percentile score (the proportion of scores below 750)?

10. **Core Concept C, Core Skill 3, Core Skill 4.**

There is little doubt that caffeine stimulates bodily activity, but how much does it take to produce a significant effect? This is a question that involves measuring the effect of two or more interventions (generally called treatments) and deciding if the different interventions have differing effects. To obtain a partial answer to the question on caffeine, it was decided to compare two levels of caffeine with a control of no caffeine on a response to a finger tapping exercise.

Thirty male students were randomly assigned to one of three treatment groups of 10 students each. Each group was given one of three doses of caffeine (0, 100, and 200 milligrams) and two hours later the students were given a finger tapping exercise. The response is the number of taps per minute, as shown in the table below.[2]

Finger taps per minute from a caffeine experiment

0 mg caffeine	100 mg caffeine	200 mg caffeine
242	248	246
245	246	248
244	245	250
248	247	252
247	248	248
248	250	250
242	247	246
244	246	248
246	243	245
242	244	250
Mean:244.8	246.4	248.3

The accompanying plot shows that the data sets tend to be somewhat symmetric and have no extreme data points (outliers) that would have undue influence on the analysis. The sample mean, then, is a suitable measure of center, and will be used as the statistic for comparing treatments.

[2]Source: Draper and Smith (1981) Applied Regression Analysis, John Wiley and Sons

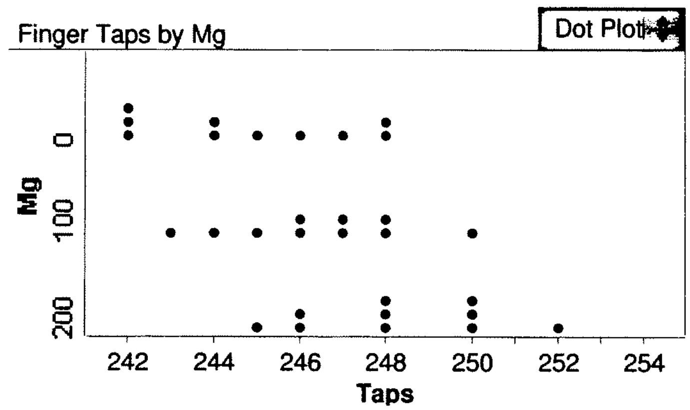

Figure 9

(a) The mean for the 100 mg data is 1.6 taps larger than that for the 0 mg data. In light of the variation in the data, is that enough to be confident that the 100 mg treatment truly results in more tapping activity than the 0 mg treatment? In other words, could this difference of 1.6 taps be explained simply by the randomization (the luck of the draw, so to speak) rather than any real difference in the treatments?

Assume for the moment that the 100 mg treatment has no effect on the tapping, and the difference in the means is simply due to the randomization. Would a difference of 1.6 units be likely to occur under these conditions? An empirical answer to this question can be found by re-randomizing the first two groups many times and studying the distribution of differences in sample means. The re-randomizing is accomplished by combining the data in the first two columns, randomly splitting them into two groups of ten each representing 0 and 100 mg, and then calculating the difference in the sample means. (This can be expedited with the use of technology.)

The plot below shows the differences produced in 200 re-randomizations of the data for 100 and 0 mg. The observed difference of 1.6 taps is equaled or exceeded 18 out of 200 times, 9% of the runs. Do these data provide strong evidence to reject the claim that the control and the 100 mg treatment do not differ with respect to their mean finger tapping counts? Explain your reasoning and the nature of the error that you may have made.

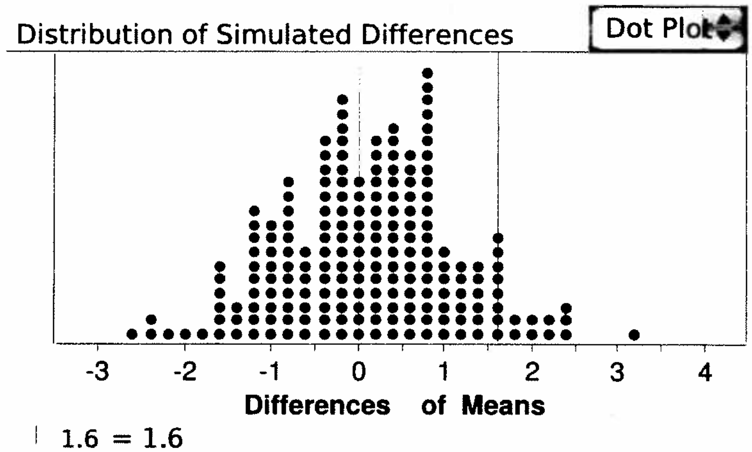

Figure 10

(b) What about the 200 mg treatment as compared to 0 mg? Construct a similar re-randomization process to decide if the difference between the mean of the 200 mg treatment and that of the 0 mg treatment is to large to be explained by chance alone. Explain your decision process and the nature of the error you may have made.

11. **Core Concept C, Core Skill 4.**

A random number generator is said to generate even or odd digits independently and each with probability 0.5. Consider the following scenarios and questions:

I. The generator generates three odd digits in a row, with probability $(.5)^3 = 0.125$ under the model given above. Does this suggest that the model is questionable?

II. The generator generates four odd digits in a row, with probability $(.5)^4 = 0.0625$ under the model given above. Does this suggest that the model is questionable?

III. The generator generates five odd digits in a row, with probability $(.5)^5 = 0.03125$ under the model given above. Does this suggest that the model is questionable?

(a) At which point in the sequence of odd digits would you begin to question the truth of the model? (If five odds in a row are not enough, continue the sequence until doubt sets in.)

(b) Write a short description explaining the interactive roles of probability and data in the decision-making process.

12. **Core Concept C, Core Skill 4.**

A random sample of 100 students from a specific high school resulted in 45% of them favoring a plan to implement block scheduling. Is it plausible that a majority of the students in the school actually favor the block schedule? Simulation can help answer the questions.

The accompanying plot shows a simulated distribution of sample proportions for samples of size 100 from a population in which 50% of the students favor the plan, and another distribution from a population in which 60% of the students favor the plan. (Each simulation contains 200 runs.) What do you conclude about the plausibility of a population proportion of 0.50 when the sample proportion is only 0.45? What about the plausibility of 0.60 for the population proportion?

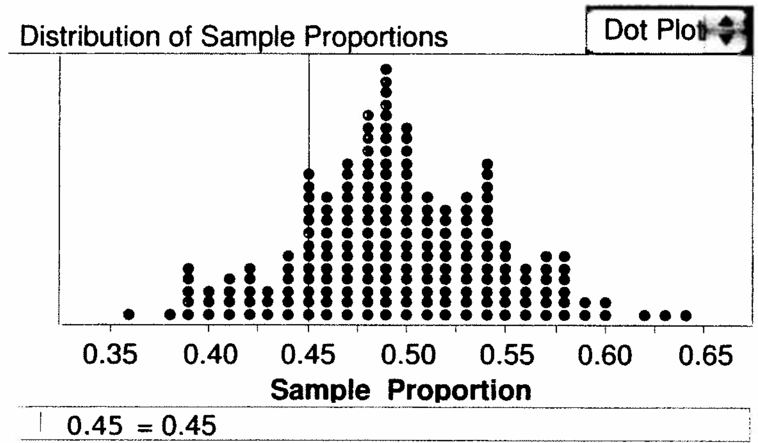

Figure 11

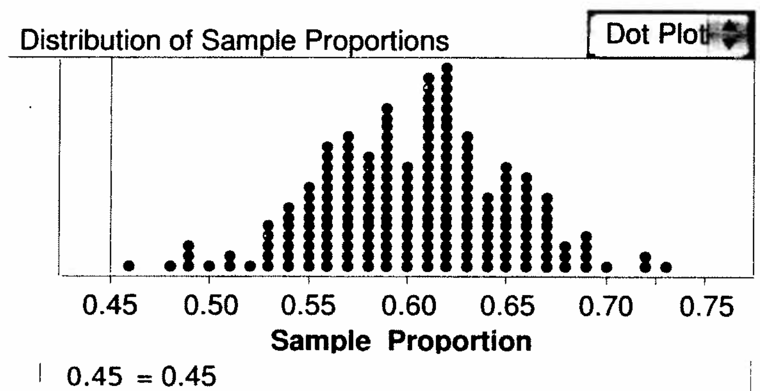

Figure 12

13. **Core Concept C, Core Skill 4.**

A tremendous amount of bottled water is being consumed these days. Do the consumers really think bottled water tastes better than city water or well water? Or, is this just a fad? To test tastes preferences in one school, $n = 50$ students were given samples of bottled water, well water from typical area homes having wells, and city water. Each student tasted all three types of water in random order. The results: 26 preferred bottled water, 16 preferred city water and 8 preferred well water.[3]

(a) Suppose the students running the experiment decided to check the claim that students had no preference for bottled water over non-bottled water. Under this claim and the randomization of the order of tasting, the preference for bottled over non-bottle water should have probability of about $p = 0.5$. The probabilities for X, the number of students choosing bottled water when $n = 50$ and $p = 0.5$, is shown below, with a vertical line at $X = 26$.

[3]Source: http://library.thinkquest.org/04apr/00222/text/survey.htm

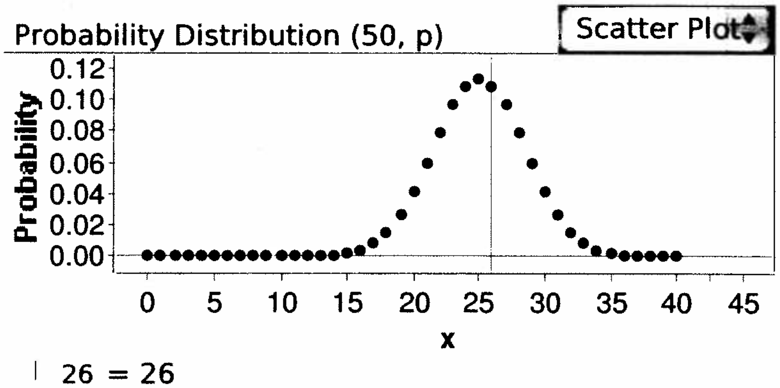

Figure 13

Does this provide strong evidence to reject the claim that there is no preference for bottled water? Explain your reasoning.

(b) Some students suggest that making this into a two-choice problem is incorrect, because there are really three choices for each taster. So, the correct claim to test is that students have no preference among bottled water, well water and city water. Under this claim the preference for bottled water should have probability of about $p = 0.33$. The probabilities for X, the number of students choosing bottled water with $n = 50$ and $p = 0.33$, is shown below, with a vertical line at $X = 26$.

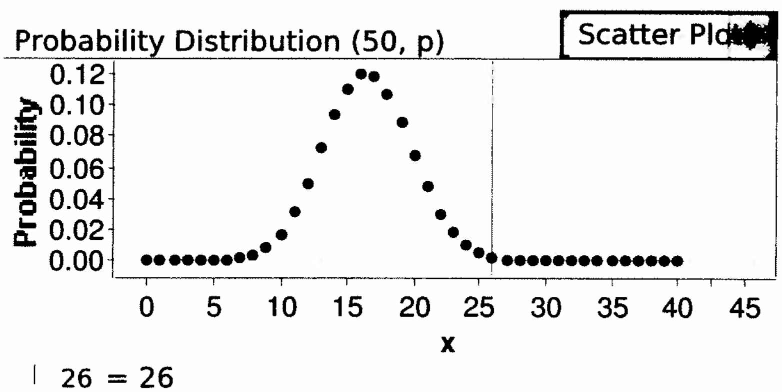

Figure 14

Does this provide strong evidence to reject the claim of no preference for bottled water among the three choices? Explain your reasoning.

(c) Which of the two claims outlined above seems more appropriate for this experiment?

(d) In the decisions made above, explain the error that could have been made.

14. **Core Concept D.**

From a class containing 12 girls and 10 boys, three students are to be selected to serve on a school disciplinary panel. Here are four different methods of making the selection.

I. Select the first three names on the class roll.
II. Select the first three students who volunteer.
III. Place the names of the 22 students in a hat, mix them thoroughly, and select three names from the mix.
IV. Select the first three students who show up for class tomorrow.

Which is the best sampling method, among these four, if you want the school panel to represent an unbiased view of the opinions of your class.

15. **Core Concept D, Core Skill 1.**

Biology students are to conduct a study of the effect of different durations of light and dark on the growth of radish seedlings. To pare the number of possible durations down to a manageable level, the students decided to focus the question on three different durations: 24 hours of light, 12 hours of light and 12 hours of darkness, and 24 hours of darkness. Plastic bags are to be used as growth chambers; there are 120 radish seeds available for the study.

(a) Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
(b) Describe how you would design a study to compare the three duration schemes, using all 120 seeds and as many plastic bags as you need.

16. **Core Concept D, Core Skill 1.**

Students in a high school mathematics class decided that their term project would be a study of the strictness of the parents or guardians of students in the school. Their goal was to estimate the proportion of students in the school who thought of their parents or guardians as "strict". They do not have time to interview all 1000 students in the school, so they plan to obtain data from a sample of students.

(a) Is the best design for this study a sample survey, an experiment, or an observational study? Explain your reasoning.
(b) The students quickly realized that, as there is no definition of "strict", they could not simply ask a student, "Are your parents or guardians strict?" Write three questions that could provide objective data related to strictness.
(c) Describe an appropriate method for obtaining a sample of 100 students, based on your answer in part (a) above.

17. **Core Skill 1, Core Skill 2.**

The data in the accompanying table shows the fate of 1316 passengers on the Titanic described in terms of two categorical variables, class of travel and survival. Use these data to answer the questions posed below.

	Class of Travel			
Survived	First	Second	Third	Total
Yes	203	118	178	499
No	122	167	528	817
Total	325	285	706	1316

(a) Do these data come from a sample survey, an experiment or an observational study? Explain your reasoning.
(b) What proportion of the passengers survived? What proportion of the passengers traveled in first class?
(c) Construct a table of (conditional) relative frequencies by column. Interpret the results in context and use them to discuss the association between class of travel and survival.
(d) Construct a table of (conditional) relative frequencies by row. Do these result in a similar interpretation with regard to association as that in part (c)?

18. **Core Skill 1, Core Skill 2, Core Skill 3.**

A simple random sample of 100 high school seniors was selected from a large school district. The gender of each student was recorded, and each student was asked the following questions.

- Have you ever had a part-time job?
- If you answered yes to the previous question, was your part-time job in the summer only?

The responses are summarized in the table below.

	Gender		
Job Experience	Male	Female	Total
Never had a part-time job	21	31	52
Had a part-time job during summer only	15	13	28
Had a part-time job but not only during summer	12	8	20
Total	48	52	100

(a) Construct a graphical display that shows the relationship between gender and job experience for the students in the sample.
(b) Write a few sentences summarizing what the display in part (a) reveals about the relationship between gender and job experience for the students in the sample.
(c) Is it appropriate to use these data to construct an estimate of the proportion of seniors in the school district who never had a part-time job? Why or why not?

19. **Core Skill 1, Core Skill 3.**

The 54 students in a middle school class were asked two questions about musical preferences: "Do you like rock?" "Do you like rap?" The responses are summarized in the table below.

(a) Is this a sample survey, an experiment, or an observational study?
(b) What percentage of the students in the class like rock?
(c) Does there appear to be a positive association between liking rock and liking rap for this class (i.e., do the students who like rock also tend to like rap)? Justify your answer by pointing out a feature of the table that supports it.
(d) Do you think the results for this class would generalize to the entire middle school? To a high school class in a nearby school? Explain your reasoning.

	Like Rap		
Like Rock	Yes	No	Row Totals
Yes	27	6	33
No	4	17	21
Column Totals	31	23	54

20. **Core Skill 1, Core Skill 5.**

What are the consequences of delayed defibrillation for those who have cardiac arrest? In a study of 6789 patients, at 369 hospitals, who had experienced cardiac arrest, 2045 had to wait longer than the recommended two minute maximum for defibrillation. Among those receiving delayed defibrillation, 22.2% survived to hospital discharge. Among those whose defibrillation was not delayed, 39.3% survived to discharge. The study concluded that, "Delayed defibrillation is common and is associated with lower rates of survival after in-hospital cardiac arrest."

(a) Is this study a sample survey, an experiment or an observational study?

(b) Note the use of the phrase "is associated with" in the study conclusion. Would it be correct to replace this phrase with "causes"? Why or why not?

21. **Core Skill 2, Core Skill 3.**

Two gasoline additives were tested to check their claims that their use will increase gasoline mileage in cars. Thirty cars were randomly selected and each was filled with gasoline. The cars were run until the gas tanks were empty. The distance traveled was recorded for each car.

Additive A was randomly assigned to 15 of the cars and the remaining 15 cars got additive B. The cars were again run under the same driving conditions until the tanks were empty and the miles driven was recorded. For each car the difference in miles driven was calculated as miles with additive minus miles without the additive. If that number was greater than zero it meant that the car went a greater distance with the additive than without it. Negative differences meant that the car went a greater distance without the additive than with it.

The following table summarizes the calculated differences.

Additive	Values Below Q1	Q1	Median	Q3	Values Above Q3
A	-10, -8, -2	1	3	4	5, 7, 9
B	-5, -3, -3	-2	1	25	35, 37, 40

(a) On the grid below, display parallel boxplots (showing outliers, if any) of the differences for the two additives. Make sure that you label the plots so that we can tell which one is A and which is B.

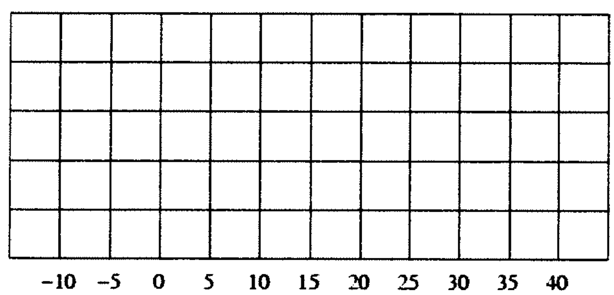

Figure 15

(b) Two ways that the effectiveness of a gasoline additive can be evaluated are by looking at

- The proportion of cars that have increased gas mileage when the additive is used in those cars
- The average mean increase in gas mileage when the additive is used in those cars.

(i) Which additive, A or B, would you recommend if the goal is to increase gas mileage in the highest proportion of cars? Explain your answer.

(ii) Which additive, A or B, would you recommend if the goal is to have the highest average (mean) increase in gas mileage? Explain your answer.

22. **Core Skill 3.**

The accompanying graph shows the relationship between scores on Exam 1 and scores on Exam 2 for all the students studying statistics from the same teacher in a recent year. The darker line (with slope 1) is the $y = x$ line. The lighter line (slope 0.6) is the least squares line of best fit.

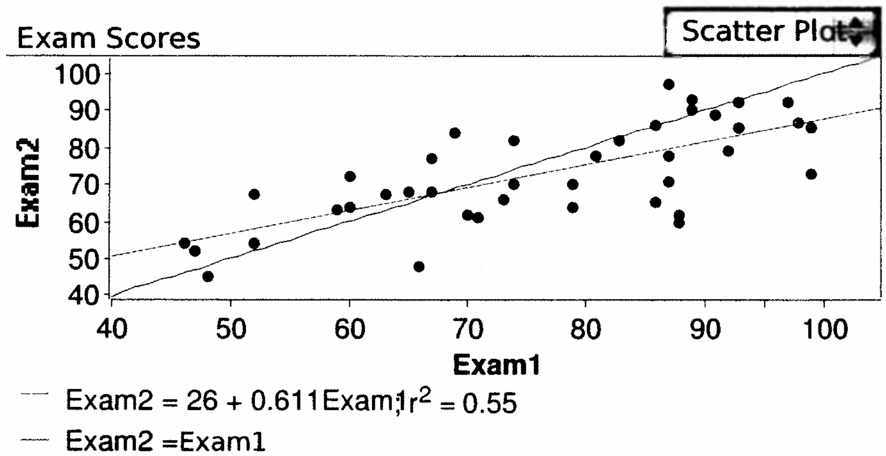

Figure 16

(a) What does the pattern tell you about the students who scored 60 or below on the first exam? What does the pattern tell you about the students who scored 80 or above on the first exam?
(b) Did more students improve their score or lower their score as they moved from exam 1 to exam 2? What feature of the plot leads to you answer of this question?
(c) Make a concise statement as to why the slope of the least squares line is less than 1.

23. **Core Skill 3.**

Gas chromatography is a technique used to detect very small amounts of a substance. To study the calibration of a gas chromatograph, five measurements were taken for each of four specimens containing different but known amounts of the substance being studied. A graph of the output readings from the gas chromatograph versus the input amounts is shown on the accompanying plot.[4]

(a) Based on just the scatter plot and the regression line, does the simple linear model appear to provide a good description of the performance of the chromatograph?
(b) On studying the residual plot, what concerns might you have about the simple linear regression line as a model for the performance of the chromatograph? (Residuals are deviations of the points from the regression line.)

[4] Source: http://lib.stat.cmu.edu/DASL/Datafiles/Chromatography.html

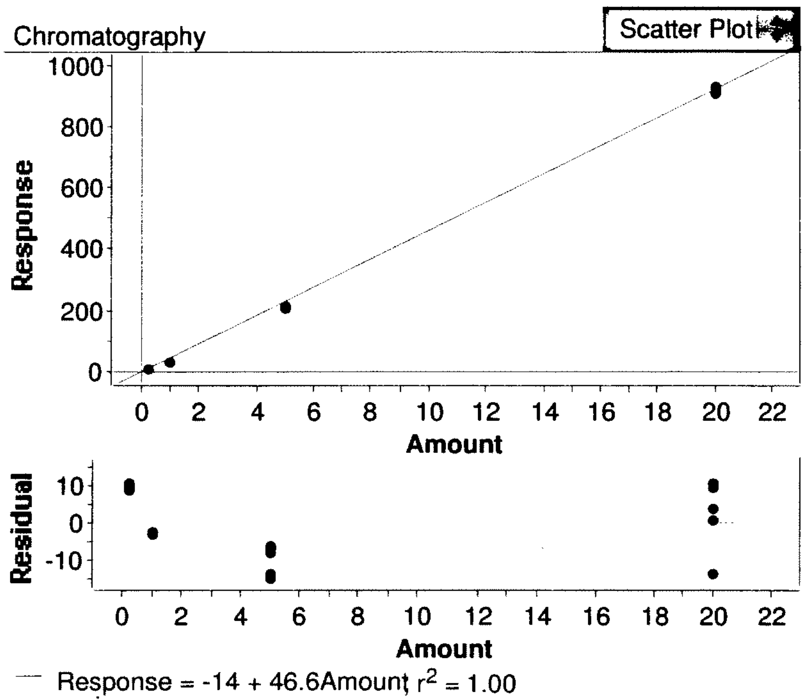

Figure 17

24. **Core Skill 3.**

Which of the following data sets are not good candidates to be modeled by a normal distribution? Explain your reasoning in each case.

(a) Salaries of employees of a large corporation.
(b) The life lengths of batteries of a specific brand used in laptop computers.
(c) Measurements of the diameters of 15-inch wheels produced by an automobile manufacturer.
(d) Prices of single-family houses in your city or county.

25. **Core Skill 3, Core Skill 4.**

Each of the simulated distributions generated above can be modeled by a normal distribution with center at the fixed population proportion, p, used to generate the distribution and standard deviation (SD) of approximately 0.05. From properties of the normal distribution it can be seen that about 95% of the possible values of a sample proportion lie within approximately 2SD of their mean (p).

(a) Letting p denote a population proportion and an observed sample proportion, argue that any population with a value of p greater than $p + 2\text{SD}$ would not produce the observed within its middle 95% of possible outcomes. (In the context of Block Scheduling, and using an arbitrary but common definition of plausible, any value of p greater than $0.45+0.10 = 0.55$ would not be a plausible value for the population proportion of students favoring the block scheduling plan.)
(b) Letting p denote a population proportion and an observed sample proportion, argue that any population with a value of p less than $p - 2\text{SD}$ would not produce the observed within its middle 95% of possible outcomes. (In the context of Block Scheduling, any value of p less than $0.45 - 0.10 = 0.35$ would not be a plausible value for the population proportion of students favoring the block scheduling plan.)
(c) Based on parts (a) and (b), argue that the interval

$$p \pm 2\text{SD}$$

forms an interval of plausible values for the true population proportion. That is, the interval $p \pm 2\text{SD}$ contains about 95% of the possible values of the sample proportions that can be generated from a population with true proportion p. This 2SD bound is commonly referred to as the margin of error.

(d) In general, will the margin of error increase or decrease as the sample size increases? Explain your reasoning.

26. **Core Skill 5.**

Researchers have noticed that the number of golf courses and the number of divorces in the United States are strongly correlated and both have been increasing over the last several decades. Can you conclude that the increasing number of golf courses is causing the number of divorces to increase? Explain your answer.

27. **Core Skill 5.**

The following appeared in the Sunday Times of London:

> Research finds the website [Facebook] is damaging students' academic performance.... Facebook users ... are more likely to perform poorly in exams, according to new research. ... The majority of students who use Facebook every day are underachieving by as much as an entire grade compared with those who shun the site.

This story was then picked up by many other newspapers and became a popular headline across the United States and beyond.

(a) What elements of the story in the Times sound like cause-and-effect arguments?

The source of the data in this case was the report of a study at the Ohio State University entitled, "A Description of Facebook Use and Academic Performance Among Undergraduate and Graduate Students." A total of 219 participants, mostly from selected classes in the school of education whose instructors agreed to participate, constituted the sample of this pilot study. The sample included both graduate and undergraduate students, with self-reported data on both how much they studied and grade point averages. The data were reported in ranges, like 3.0 to 3.5 for grade point averages. [5]

(b) Many of the weaknesses of the study were reported by the authors. What do you think some of these weaknesses are?

(c) Was the Sunday Times correct in reporting the results in cause-and-effect style? Explain your reasoning.

28. **Core Skill 5.**

A presidential campaign advertisement for former New York City mayor Rudy Giuliani said, "I had prostate cancer, 5, 6 years ago. My chance of surviving prostate cancer-and thank God, I was cured of it-in the United States was eighty-two percent. My chance of surviving prostate cancer in England was only 44 percent under socialized medicine."

The quoted percentages were based on 5-year survival rates, not mortality rates, reported from studies in the United States and the United Kingdom, respectively. But, these 5-year survival rates have little to do with mortality rates, as can be seen from the definitions given below:

$$\text{5-year survival rate} = \frac{\text{number of patients diagnosed with cancer still alive 5 years after diagnosis}}{\text{number of patients diagnosed with cancer}}$$

$$\text{annual mortality rate} = \frac{\text{number of people who died from cancer over 1 year}}{\text{number of people in the group}}$$

(a) How could changes in medical practice increase the 5-year survival rates without changing the mortality rate for any given year?

[5] Source:http://online.wsj.com/article/SB124034974305240495.html

(b) In the US, most prostate cancer is detected by screening for prostate-specific antigens (PSA). In the UK, most prostate cancer is detected by symptoms. How might this difference in medical practice affect their respective 5-year survival rates?
(c) Was Mr. Giuliani making a fair comparison? Explain your reasoning.

29. **Core Skill 5.**

"Parents Rate Schools Much Higher Than Do Americans Overall" is the title of a report on a Gallup poll of August 24, 2009. The results of the poll show that "three in four American parents (76%) are satisfied with the education their children receive in school, compared to 45% of the general public who are satisfied with the state of schools nationwide." An explanation of how the poll was conducted includes the following.

> Results are based on telephone interviews with 1,010 national adults, aged 18 and older, conducted Aug. 6-9, 2009. For results based on the total sample of national adults, one can say with 95% confidence that the margin of error is ±4 percentage points. For results based on the sample of 233 parents with children in kindergarten through grade 12, the maximum margin of sampling error is ±8 percentage points.
>
> Interviews are conducted with respondents on land-line telephones (for respondents with a land-line telephone) and cellular phones (for respondents who are cell-phone only).
>
> In addition to sampling error, question wording and practical difficulties in conducting surveys can introduce error or bias into the findings of public opinion polls.

(a) Explain the meaning of the 4% margin of error in the context of this poll.
(b) Why is the margin of error for the poll of parents with children in school larger than that for the overall poll?
(c) Why is it important to mention both cellular and land-line phones?
(d) Other than error due to the fact the data comes from a sample, describe other potential sources of error in a poll of this nature.

30. **Core Skill 5.**

In 1962, research statistics showed that the percentage of obesity in America's population was at 13%. By 1980 it has risen to 15%, by 1994 to 23%, and by the year 2000 the obesity progression in America had reached an unprecedented 31%. The U.S. Surgeon General report declared that obesity is responsible for 300,000 deaths every year. These overwhelming research statistics reveal an alarming obesity trend, the need for diagnosis, and a call to action.

In the midst of an informational and research feeding frenzy on the obesity epidemic, statistics are easy to come by. The most widely disseminated CDC research statistics on American obesity tell us that 63% of adult Americans have a Body Mass Index (BMI) in excess of 25.0 and are therefore overweight; more than a quarter surpass 30.0, having been declared obese. And perhaps the most riveting statistics concern obesity in kids: research shows that childhood obesity has more than tripled over the past two decades.

Still, by focusing on the polite abstraction of Body Mass Index rather than actual bodyweight, the CDC has hindered the war on the prevention of obesity in America. Indeed, CDC research epidemiologists are faithful keepers of the public health record; but for reasons of technical precision and political propriety, they have scrupulously avoided the publication of the most crucial and powerful obesity statistics, raw bodyweight averages for the American population.

The new IHRSA/ASD Obesity/Weight Control Report has published these graphic and visceral images of a dangerously overweight population. The "real" research statistics on obesity reveal that:

- 3.8 million Americans carry over 300 pounds.
- With the average adult woman weighing in at a staggering 163!

Perhaps the most shocking statistics underscoring obesity in the United States is that 400,000 Americans (mostly men) fall into a super-massive 400+ pound category.[6]

(a) Explain the controversy between the two measures of obesity, the BMI (which takes into account height and weight) and bodyweight averages.

(b) The percentages in the first paragraph are based on BMI. Is there any other data on the article to which these figures can be compared fairly?

(c) Does the article make a strong scientific case for using bodyweight rather than BMI to measure obesity?

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Exhibit C-4

NATIONAL GOVERNORS ASSOCIATION

CCSSO
THE COUNCIL OF CHIEF STATE SCHOOL OFFICERS

Common Core State Standards Initiative

Summary of Public Feedback on the Draft College- and Career- Readiness Standards for English-Language Arts and Mathematics

The point of the state-led effort to create common academic standards is simple: improving teaching and learning to ensure that high school graduates in *every* part of the nation have the knowledge and skills they need for college or a career. The process is designed to produce standards that are research and evidence-based as well as internationally benchmarked. If students meet these new rigorous and clear standards, they will have better choices in their lives and the nation will be more competitive in today's global economy.

State leaders clearly understand that these common academic expectations are the essential building block to significantly improve education for all students. They are also listening carefully to a variety of audiences to make sure the new Common Core State Standards provide the excellence and clarity that educators and students require. To this end, a draft of the common core standards was available for public comment between September 21 and October 21, 2009.

Below, the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO) provide highlights from the public feedback on the draft college- and career-readiness Common Core State Standards for English-language arts and mathematics.

Background

This summer, two work teams—one for English-language arts (ELA) and one for mathematics—created the first drafts of the college- and career-readiness Common Core State Standards (CCSS). Team members were selected for their content knowledge and extensive experience with standards. Many of the work team members began their careers as classroom teachers and have extensive experience setting standards and aligned assessments.

Using an iterative process, the organizations have shared ever-stronger drafts with larger and larger audiences. Please click here to view a graphic display of the standards development process. The major steps include:

- Work teams create first draft standards in early summer;
- Content experts (for example, mathematicians and mathematics educators reviewed the mathematics standards) examine first draft standards;
- Working teams use comments to revise standards during July 2009;
- States and national organizations review and comment on second draft of the standards during August 2009;

- Work teams use feedback to revise standards;
- Drafts made available for public comment during September and October 2009 (a summary of the feedback begins on page 2 of this report);
- The process continues, with K-12 Work and Feedback Teams soliciting and considering state, national organization, and public feedback as they finalize the college- and career-readiness standards and develop the K-12 standards; and
- A Validation Committee provides advice to the entire process and products of the initiative.

Highlights of Public Feedback

The feedback represents the opinions of more than 1,000 people. There were 988 online surveys completed via www.corestandards.org, and in many cases, a single response represents the input of multiple individuals. In some cases, people worked together to submit feedback, and multiple names are listed on a single survey response. Other times, an authorized person submitted a response representative of the opinions of hundreds of members of an organization. In addition, some organizations convened focus groups for the purpose of soliciting opinions, which were submitted by the organization as a single, synthesized response. And finally, beyond the online submissions and not reflected in those numbers, about three dozen groups or individuals submitted feedback directly to NGA or CCSSO.

Who Responded?
The respondents self-identified and were able to select multiple categories. Keeping in mind that a single one of the 988 respondents might actually represent many people, it is not always possible to know whether a respondent considers herself both a teacher and a professor or whether two people worked jointly to complete the survey. With these caveats, the respondents identified themselves in the following categories:

- 53 percent as educators and another category;
- 29 percent as content experts;
- 28 percent as teachers;
- 22 percent as parents and another category;
- 3 percent as parents only;
- 14 percent as professors;
- 10 percent as school district staff;
- 8 percent as students and another category;
- 1 percent as students; and
- 5 percent as state education agency staff.

(The percentages exceed 100 percent because people identified themselves in multiple categories.)

What Types of Comments Were Submitted?

Survey respondents were given the option to submit comments specific to particular ELA standards, mathematics standards or general comments on either set of standards. Most comments submitted were overall or general comments. To summarize, there were:

- 255 general comments about the ELA standards;
- 87 comments on one or more specific ELA standards;
- 169 general comments about the mathematics standards;
- 96 comments on one or more specific mathematics standards; and
- 359 respondents commented on some element of both the ELA and mathematics standards.

Themes Across Content Areas

Most of the comments were specific to either mathematics or ELA, but a few topics surfaced in the comments in both content areas.

The Importance of a Standards-Based System

Many respondents said that while it is important to get the standards right, standards are only one part of a complex system. Respondents said it is important to build an assessment system that measures fairly what is important and gives teachers timely information. They spoke to the necessity of having a robust curriculum that is aligned to the standards and also allows individuals to maximize their potential beyond meeting the standards. Respondents stressed the need for professional development designed to better equip the existing teaching force and for teacher preparation programs that seek out the best and brightest candidates, prepare them well, and support them as they start teaching.

Some respondents took the opportunity to share grievances and concerns not specific to either content area and beyond the scope of the Common Core State Standards Initiative. Most of these are isolated comments on a unique topic.

Anti-Standard or Anti-Testing Sentiments

There was no statement directly against the Common Core State Standards, but a few respondents expressed the idea that a focus on standards comes at the expense of developing individual students. It was also stated that standards do little to improve education but rather drain money away from the classroom and feed bureaucracy.

Standards Writing Process

Some respondents questioned the process used to draft the standards. There were comments about whether the process could have been more transparent and complaints that too few classroom teachers were involved in the writing process. There were also concerns about possible conflicts of interest because some team members are associated with the testing industry.

Reactions to the College- and Career-Readiness English-Language Arts Standards

A number of respondents, most often classroom teachers, found the standards to be exemplary. As one teacher states, "These standards are excellent! They are relevant, realistic, and rigorous. The format of the document itself is user-friendly; this is great work! Thank you!"

The majority of respondents reacted favorably to the standards but want adjustments.

"Less is More" versus "More is Better"

Respondents conceptually embrace the idea of "fewer, clearer, higher" standards. However, most also suggest the standards be expanded in one or more areas. Respondents suggested dozens of topics that could also be added or expanded, but rarely is it suggested that a topic be eliminated or minimized. Among the topics suggested to be added to the standards are 1) civic readiness; 2) applied learning; 3) awareness of author strategies; 4) collaboration; 5) oral and written language development specific to disciplines; 6) the way that gender, race, class, and culture shape our textual interpretations; 7) ability to navigate in a digital world; 8) differences in formal and information rules among forms of genres; 9) topics and research questions; 10) flexible writing processes; 11) reading for pleasure; 12) viewing skills; and 13) vocabulary development.

There is a predictable relationship between a respondent's expertise and his or her suggestions. Writing teachers want more specificity about the process, types, and purposes of writing woven into the Common Core State Standards; librarians tend to be more sensitive to the opportunities and demands created by the online environment; and reading teachers offer much more detailed and specific standards related to teaching reading.

Make the Study of Literature and a Defined Reading List Explicit

A number of respondents cited two connected additions to the standards: the importance of having a literature standard and the importance of including a defined reading list. Respondents believe that it is through literature that students come to understand the possibilities of language, gain access to the major genres, find models of style and syntax for their own writing, and develop historical and philosophical knowledge. Some respondents believe that defining great books that all students should read is a core piece of this teaching. They express a concern that the emphasis of the draft standards is reductive and too focused on the workplace.

Reactions to the Mathematics Standards

The reactions to the draft mathematics standards are generally positive. Although respondents identified many items to adjust, this seems to be a standards document most respondents find acceptable. For a few, either the idea of common state standards or the Common Core document itself unleashed uncontrolled enthusiasm. In the words of one teacher, "THANK YOU for creating standards! I'm a math teacher, and am thrilled to be held to the same standard as other math teachers in other states. Thank you, thank you, thank you..."

Differing Opinions about the Content to Include

The theme around which there seems to be the most tension is in trying to sort out how much math it really takes to be prepared for life beyond high school and the nature of that math. Defining exactly what content prepares students for entry-level, credit-bearing college courses and workforce training programs and does not unduly burden other students is complicated.

Opinions divide along the lines of those who are connected to higher education and those who are not. The respondents who teach at the college level indicated that the standards lack key content, including 1) solutions of systems of linear equations with two or more variables using determinants; 2) solutions of systems of quadratic equations; 3) exponential equations; 4) logarithms; 5) solution of polynomial equations; 6) binomial theorem; 7) permutations and combinations; 8) trigonometric functions and identities; 9) analytic geometry; 10) analytic geometry (distance formula, midpoint formula, translation of axes, distance from point to a line); 11) parametric and polar equations; 12) conic sections (equations for parabola, circle, ellipse, hyperbola); and 13) complex numbers.

Other respondents, including high school teachers and those who work in vocational fields, see the content in the standards and model problems to be well beyond what is needed by work-bound students or those going on to non-technical study at the college level. Specific examples of content suggested for removal from the standards includes 1) completing the square, 2) graphing linear inequalities with two variables, 3) solving sets of equations with three variables, 4) conditional probability, and 5) modeling using probability and statistics.

Standards Organization

There were also many comments on the organization of the standards. Many respondents are pleased with the current organization of the document, and those who are not disagree on how it should be improved.

Comments include concerns that the 10 content standards form artificial breaks in the subject of mathematics; that the topics of mathematics would be better served if some of the standards were collapsed; that some topics are actually subtopics given artificial status by the current organization; and that some strand names should be changed to better reflect the content.

There is also concern that the standards document does not indicate which topics deserve priority because of their importance. Some respondents worry that the document suggests a false priority because different topics are written at different levels of specificity. This means that for some topics there are numerous standards that over emphasize the amount of content connected to the standards. Intended or not, the absence or presence of detail establishes priorities.

Concerns with Problems

Respondents felt that the example problems are central to conveying the intentions of the standards. There were many comments on the example problems, including 1) comments related to the level of rigor the problems represent, 2) suggestions for improving the both the existing problems, and 3) a call to increase the pool of problems available. There is confusion around the purpose the example problems should serve. Most people see them as illustrative, but more than a few respondents questioned a problem's suitability for a high-stakes assessment environment.

The overriding theme is that respondents want more and better example problems, available in multiple formats.

Consistency and Accuracy

Another theme is about consistency and accuracy. Respondents described what they perceived as mathematical flaws, imprecise language, and slips in the internal consistency of the document.

Respondents identify instances when mathematical aspects of the standards lack the appropriate degree of precision. Some comments referred to an erroneous or too-broad use of a mathematical term, some comments offered a more stringent definition of a mathematical phrase, and some comments identified inconsistent uses of the same term.

Respondents noted that the structure of the document is not completely parallel. For example, the names of the strands are different types of things. "Statistics" is an area of study; "Modeling" can be described as "how one does mathematics;" and "Functions" is a basic concept in the field of mathematics. In another example, the strands do not have the same supporting elements. For a case in point, the standards document states that it looks to future topics, but only in the area of "Probability" are those topics actually stated.